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Strengthening early mathematics in early childhood classrooms : a professional development project

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Strengthening early mathematics in early childhood classrooms : a professional development project

Abstract

This project outlines a series of professional development sessions for preschool teachers that focus on early mathematics. Each session emphasizes key mathematical content and builds teachers' understanding and positive attitudes toward mathematics. Teachers also explore how children's knowledge and skills in the content topics develops and examine examples of children at various levels of understanding and stages of learning with regard to mathematical content.

The sessions also emphasize how the mathematical content can be explored in preschool classrooms in ways that support children's foundational learning in the area of mathematics. Another element of this project is the use of site-based coaching to support teachers as they implement the knowledge and skills gained into learning experiences for children in their classrooms.

Strengthening Early Mathematics in Early Childhood Classrooms:

A Professional Development Project

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ABSTRACT

This project outlines a series of professional development sessions for preschool teachers that focus on early mathematics. Each session emphasizes key mathematical content and builds teachers' understanding and positive attitudes toward mathematics. Teachers also explore how children's knowledge and skills in the content topics develops and examine examples of children at various levels of understanding and stages of learning with regard to mathematical content. The sessions also emphasize how the mathematical content can be explored in preschool classrooms in ways that support children's foundational learning in the area of mathematics. Another element of this project is the use of site-based coaching to support teachers as they implement the knowledge and skills gained into learning experiences for children in their classrooms.

TABLE OF CONTENTS

INTRODUCTION

For my graduate project I designed a professional development course for preschool teachers to develop their knowledge of key early mathematics skills and support the implementation of this knowledge into daily practice within early childhood classrooms through the intentional teaching of important mathematics content.

Rationale

There are a number of reasons why I chose to develop this project. As a consultant with an Area Education Agency, one of my responsibilities is to develop and deliver professional development for preschool teachers in our area. While mathematics has been woven into various professional development sessions over the years, there have not been any courses offered in which early mathematics has been the primary focus.

Over the past several years, there has been a great deal of emphasis placed on literacy in classrooms across the State of Iowa. The focus on literacy has been amplified by the adoption of Iowa Code section 279.68 or what has come to be known as the Early Literacy Implementation (ELI) law addressing reading proficiency. In response, school districts across the State are examining the instruction and assessment of literacy skills in the early elementary grades. While preschool is not included in ELI, preschool teachers are feeling the urgency of developing children's foundational literacy skills. As a result, literacy opportunities receive top priority over mathematics in preschool classrooms. A number of teachers have shared with me their personal frustration with the fact that mathematics is being pushed aside.

In the fall of 2014, a team of consultants within our agency began to examine the condition of mathematics teaching and learning in early childhood classrooms in our area.

Consultants reported a great deal of variance in the mathematics opportunities observed in preschool classrooms. A review of data from the Teaching Strategies GOLD Assessment (Heroman, Burts, Berke, & Bickart, 2010) revealed more than 50% of the children enrolled in a Statewide Voluntary Preschool Program demonstrated knowledge and skills that were below widely held expectations in the area of mathematics. Recognizing the need to increase the quality of mathematics teaching and learning in preschool classrooms, I attended the Early Math Summer Institute at the Erikson Institute in Chicago in July 2015 with a team of colleagues. The Institute focused on the work of the Erikson Institute Early Math Collaborative and highlighted the foundational skills that should be developed during the preschool years and methods for incorporating these skills into early childhood classrooms. All of these circumstances brought me to the realization that it is the appropriate time to develop a course for preschool teachers that is focused on the topic of early mathematics.

Importance of Project

In 2002, a joint position statement developed by the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM) highlighted the importance of incorporating high quality mathematics experiences into preschool classrooms saying, "high-quality, challenging, and accessible mathematics education for 3- to 6-year-old children is a vital foundation for future mathematics learning" (p. 1). The position statement also asserted, "In every early childhood setting, children should experience effective, research-based curriculum and teaching practices" (p. 1). The position statement went on to outline the need for strong foundational skills in mathematics, the challenges in mathematics achievement, the NCTM principles for school mathematics, and recommendations to guide classroom applications as well as policies and systems work in the area of early mathematics.

In 2009, the National Research Council (NRC) Committee on Early Childhood Mathematics released a report that summarized the findings of an extensive review of the research on mathematics in early childhood. The report presented the mathematical concepts that are appropriate in the context of early childhood settings and described factors that contribute to children's mathematical learning and the barriers that impede children's learning. The report presented teaching-learning paths which were defined as "the significant steps in learning in a particular topic; each new step in the learning path builds on the earlier steps" (p. 121). The work of the Committee on Early Childhood Mathematics resulted in the development of several recommendations for both practice and policy in this area. The report echoed the position of NAEYC & NCTM (2002) emphasizing the importance of high-quality mathematics curriculum and instruction in all early childhood programs. The overarching recommendation in the report highlighted the need for a coordinated effort to improve teaching and learning in mathematics for young children. In addition, the report offered several recommendations that addressed the alignment of standards, curriculum, instruction, and materials that support mathematics teaching and learning in early childhood programs. Finally, the report called attention to the need for professional development for early childhood educators that supports teaching and learning of mathematics.

The importance of this project is demonstrated through its alignment to the recommendations set forth in the NAEYC & NCTM (2002) position statement. This project highlights key early mathematics knowledge and skills that children should have access to in early childhood programs. In addition, this project includes classroom practices that encompass the recommendations in the position statement for high-quality early mathematics education. When teachers have a better understanding of foundational early mathematics knowledge and skills and the practices that support the development of them, they are able to intentionally plan and deliver learning opportunities that engage children in the acquisition of the development of those skills. This project also incorporates the alignment of standards, curriculum, and assessment to support mathematics teaching and learning. This project fulfills a need that exists for professional development that is focused on supporting mathematics learning in early childhood settings.

Purpose of Project

The purpose of this project was to develop a professional development course for preschool teachers that focuses on the topic of early mathematics. The course emphasizes the value of providing learning opportunities in early childhood classrooms that develop children's foundational mathematics knowledge and skills. The project mirrors the "Whole Teacher Approach" used in the Early Mathematics Education (EME) Project at the Erikson Institute with a goal of positively impacting teachers' knowledge, attitudes, and practices with regard to early mathematics (Chen & McCray, 2012). The course highlights the big ideas of early mathematics (Clements & Sarama, 2014; Erikson Institute Early Math Collaborative, 2014). Clements and Sarama (2014) described big ideas of mathematics as, "clusters of concepts and skills that are mathematically central and coherent, consistent with children's thinking, and generative of future learning" (p. 3). Erikson Institute Early Math Collaborative (2014) echoes this definition of the big ideas. The big ideas are reflective of the Common Core State Standards and the work of NCTM (Clements & Sarama, 2014; Erikson Institute Early Math Collaborative, 2014). The course increases teachers' knowledge of foundational early mathematics skills and instructional practices to intentionally integrate mathematics learning into activities and routines within early childhood classrooms. The course also develops teachers' understanding of how the integration of mathematics learning in preschool classrooms connects to the standards and objectives that guide early childhood programs in Iowa including the Iowa Early Learning Standards (Early Childhood Iowa, 2012), the Iowa Quality Preschool Program Standards (Iowa Department of Education, 2006), and the Creative Curriculum/Teaching Strategies GOLD Objectives for Development and Learning (Heroman, Burts, Berke, & Bickart, 2010). Finally, this project includes on-site coaching with preschool teachers to support the implementation of knowledge and skills acquired during the professional development sessions into classroom practice.

METHODOLOGY

This project incorporates the Iowa Professional Development Model (IPDM) by focusing on student learning and engaging educators in focused and collaborative professional development (Iowa Department of Education, n.d.). A critical element of IPDM is that professional development is not a one-time event. It is an ongoing process of action research because, "If new content is to be learned and implemented in classrooms so that students benefit, teachers need ongoing training, the colleagueship of peers as they plan and develop lessons and materials and study their implementation, and interim measures to judge the success of their efforts" (Iowa Department of Education, n.d., "The Cycle of Professional Development," para. 5). These beliefs were central to the development of this course.

Course Content

The content in this professional development project highlights the importance of developing children's early mathematics knowledge and skills in the preschool years. The foundational mathematics knowledge and skills highlighted in the big ideas of mathematics will be the primary focus of the content in the course. Topics include sets and sorting, number sense, counting, number operations, patterns, measurement, data analysis, spatial relationships, and shapes (Clements & Sarama, 2014; Erikson Institute Early Math Collaborative, 2014). The content also focuses on how the big ideas align with the Iowa Early Learning Standards (Early Childhood Iowa, 2012), the objectives for development and learning in the Creative Curriculum for Preschool and the Teaching Strategies GOLD assessment (Heroman et al., 2010), as well as the NAEYC Early Childhood Program Standards and Accreditation Criteria (NAEYC, 2015) and the Iowa Quality Preschool Program Standards (Iowa Department of Education, 2006). While increasing teacher knowledge of the big ideas is an important element of this project, it is also important to assist teachers in translating the content knowledge into classroom practice. Therefore, it is important that the course content highlights strategies for incorporating mathematics into the routines and activities throughout the preschool day. The recommendations presented in the NAEYC/NCTM position statement (2002) and Teaching Math to Young Children (Frye et al., 2013) are included as well as research-based practices outlined (Clements & Sarama, 2011; Erikson Institute Early Math Collaborative, 2014; Fuson et al., 2015; Jung & Conderman, 2013; Klibanoff, Levine, Huttenlocker, Vasilyeva, & Hedges, 2006; Linder, Powers-Costello, & Stegelin, 2011; Rudd, Satterwhite, & Lambert, 2010). Several sessions connect to classroom practices by exploring the mathematics in blocks, routines, games, and books.

Delivery of Course Content

The target audience for this project is teachers in district or community preschool programs serving three-, four- and five-year-olds. The content of this course was designed to be delivered in a series of 9 half-day professional development sessions spaced out over the course of the school year to allow for coaching and classroom implementation between sessions. The sessions focus on increasing teachers' knowledge of the big ideas through experiences that are both participatory and collaborative. Engaging teachers in an experience in which they view themselves as learners results in changes in their attitudes toward mathematics. The collaborative nature of the sessions also creates a supportive climate for teachers in which they are able to share ideas, problem solve, and celebrate successes experienced in their classrooms (Chen & McCray, 2012).

Course Materials

The materials developed for this course include presentation slides, hands-on learning activities, and participant handouts to be used in the delivery of the course (see Appendix B – J). Several journal articles were selected for activities throughout the course. Additional course materials developed include a survey to be used as a pre- and post-test measure of participant knowledge and attitudes regarding mathematics (see Appendix M) as well as an evaluation of participant satisfaction with the course (see Appendix N). Lists of professional resources and children's books highlighting early mathematics were also developed (see Appendix K & L).

Course Follow-up

The project includes ongoing site-based coaching to support implementation of teachers' new learning into classroom practice between professional development sessions. The site-based coaching is provided by early childhood consultants and focuses on planning, observation, and reflection on mathematics activities that teachers implement in their classrooms.

LITERATURE REVIEW

Importance of Mathematics in Early Childhood

There has been increased emphasis on the importance of mathematics in the early childhood years in recent years. Duncan et al. (2007) highlighted the importance of supporting learning in the area of mathematics. In a meta-analysis of data sets from several longitudinal studies, children's mathematics knowledge and skills were found to be predictive of later achievement, not only in the area of mathematics, but also in the area of literacy. National organizations recognize the importance of early mathematics and have established recommendations that center on the need for improved early mathematics opportunities for young children (NAEYC & NCTM, 2002; NRC, 2009). The National Research Council (2001) highlighted differences in reading and mathematics that stressed the importance of school-based instruction saying,

> School-based instruction may play a larger role in most children's mathematical experience than it does in their reading experience. If so, the consequences of good or poor mathematics instruction may have an even greater effect on children's proficiency than is the case with reading. (pp. 19-20)

Despite the emphasis on the importance of supporting children's foundational mathematics learning during the preschool years, teachers spend little time intentionally teaching these skills or fail to provide mathematics instruction that is developmentally appropriate for young children (Clements & Sarama, 2011; Lewis Presser, Clements, Ginsburg, & Ertle, 2015; NAEYC & NCTM, 2002; NRC, 2009).

Teachers' Knowledge and Attitudes Regarding Mathematics

In 1986, Lee S. Shulman presented the concept of pedagogical content knowledge (PCK). It was summarized by McCray and Chen (2012) as "the coalescence of three types of knowledge necessary for effective instruction: knowledge of content, of teaching practice, and of student development" (pp. 294-295). They elaborated on the concept saying,

> PCK emphasizes (1) knowledge of which content ideas are most central and how they connect to one another (subject matter understanding), (2) appropriate examples and strategies for illustrating those concepts (teaching techniques for the subject matter), and (3) awareness of how these concepts develop in the thinking of novices with differing levels of experience (knowledge of the development of student understanding of the subject matter). (p. 295)

Several studies have explored teachers' PCK in relationship to mathematics. These studies indicated that teachers' mathematical content knowledge impacted their attitudes towards mathematics, their beliefs regarding their own ability to teach mathematics, and their ability to maximize opportunities to support children's mathematical learning (Anders & Rossbach, 2015; Bates, Latham, & Kim, 2011, 2013; Dunekacke, Jenßen, & Blömeke, 2015; Geist, 2015). Chen, McCray, Adams, and Leow (2014) examined teachers' beliefs regarding preschool and mathematics in relationship to their confidence in teaching mathematics in preschool as well as their confidence in their own mathematics abilities. Results indicated that teachers felt confident in teaching mathematics despite low confidence in their own abilities. The literature also showed teachers' knowledge and confidence varied depending on the specific mathematical content area (Chen, McCray, Adams, & Leow, 2014; Lee, 2010). These results suggest a need to examine professional development opportunities focused on mathematics for early childhood teachers.

Best Practices for Teaching Mathematical Concepts in Preschool

The literature focused on teaching mathematics in early childhood programs includes overarching best practices in the field. Mathematics education in early childhood programs involves taking advantage of opportunities to "mathematize" daily experiences that encourage children's understanding of mathematics concepts (Clements & Sarama, 2011; Erikson Institute Early Math Collaborative, 2014; Fuson, Clements, & Sarama, 2015; Lewis Presser et al., 2015; NAEYC & NCTM, 2002). In the literature, the term "mathematize" is used to describe the practice of engaging children in the mathematics in the world around them (Clements & Sarama, 2011, 2014; Erikson Institute Early Math Collaborative, 2014; Frye et al., 2013; Fuson et al., 2015; NAEYC & NCTM, 2002).

In addition to taking advantage of naturally occurring experiences, it is essential that teachers intentionally plan and carry out activities that support children's mathematical learning (Clements & Sarama, 2011; Jung & Conderman, 2013; Lewis Presser et al., 2015; NAEYC & NCTM, 2002). Lewis Presser et al. (2015) expressed this idea saying,

> During informal mathematical learning experiences, young children are motivated to learn and are already acquiring some real mathematical skills and ideas, but to advance the foundational mathematical thinking and skills required for success in

> the early grades requires deliberate teaching on the part of adults in the preschool years. (pp. 404-405)

The concept of intentionality is central to the literature on fostering mathematics in early childhood. Epstein (2014) described it as "acting purposefully." She described an intentional teacher as one who "aims at clearly defined learning objectives for children, employs instructional strategies likely to help children achieve the objectives, and continually assesses progress and adjusts the strategies based on that assessment" (p. 5). Clements and Sarama (2011) and the Erikson Institute Early Math Collaborative (2014) stressed the idea that mathematics learning requires more than simply having access to math manipulatives and materials. One study (Lewis Presser et al., 2015) emphasized this saying,

> Children often need more than their natural, spontaneous learning. In other words, play is not enough to optimize children's mathematical development. Instead, they need intentional teaching that supports learning experiences that expose them to mathematical concepts in a progressive and developmental fashion. (p. 402)

In order for children to acquire mathematical knowledge and skills, the adults in their lives need to intentionally plan for, deliver, and adjust daily opportunities that support the development of knowledge and skills (Clements & Sarama, 2011; Erikson Institute Early Math Collaborative, 2014; Frye et al., 2013; Fuson et al., 2015; Jung & Conderman, 2013; NAEYC & NCTM, 2002). Lewis Presser et al. (2015) summarized this saying, "Thoughtfully designed and executed activities can help to further develop the mathematical understanding children already have and to challenge their misconceptions" (p. 405).

Children's learning results from making mathematics concrete and multisensory to meet the needs of all learners (Erikson Institute Early Math Collaborative, 2014).

Professional Development for Early Childhood Mathematics

A review of the literature stressed the need for professional development in the area of early childhood mathematics. Lee (2010) and Lewis Presser et al. (2015) emphasized the need for professional development for teachers in order to prepare them to support young children in developing their knowledge and skills in the area of mathematics. Thornton, Crim and Hawkins (2009) highlighted two elements that should be addressed in professional development saying,

> Early childhood educators must not only be knowledgeable about mathematical concepts themselves, they must also understand the appropriate concepts to teach at each grade level and be aware of the most developmentally appropriate ways in which to teach mathematical concepts to young children. (p. 151)

However, the literature indicated professional development focused on mathematics is lacking for early childhood teachers (NAEYC & NCTM, 2002; Simpson & Linder, 2014). Simpson and Linder (2014) reported that, although professional development on mathematics was desired, it was either not offered or was insufficient because it did not adequately prepare educators to implement effective mathematics experiences in their classrooms.

Recent research highlights features of high quality professional development experiences. Professional development should be ongoing, focused on content including essential mathematical learning and how children think about mathematics, highlight research based instructional practices, result in student learning, and encourage collaborative learning among teachers (Linder, 2012; NAEYC & NCTM, 2002; National Research Council, 2009; Thornton, Crim, & Hawkins, 2009). The literature indicates successful professional development projects include an element of on-site coaching to support implementation (Chen & McCray, 2012; Clements, Sarama, Wolfe, & Spitler, 2015; Rudd, Lambert, Satterwhite, & Smith, 2009; Thornton et al., 2009). In site-based coaching, teachers are able to plan, problem solve, and reflect on their implementation of classroom practices that support children's mathematics learning. This type of ongoing support is essential to sustain changes in teacher practice and support the implementation of new learning.

THE PROJECT

This project centered on the development of a series of professional development sessions that address the why, the what, and the how to infuse mathematical learning in early childhood settings. The NAEYC & NCTM Position Statement (2002) is included in the course to emphasize the importance of this work. The content presented in each session focuses on key foundational mathematics skills presented in *Big Ideas of Early Mathematics: What Teachers of Young Children Need to Know* (Erikson Institute Early Math Collaborative, 2014). The activities and discussions in the course are designed to address three areas that impact teaching and learning of mathematics in early child classrooms: teacher attitudes and beliefs, teacher knowledge and skills, and classroom practices (Chen & McCray, 2012). The activities promote a more positive disposition toward the mathematics that is all around and engage the participants in constructing their own understanding of the targeted big ideas. The presentation and discussion of the big ideas and observation and discussion regarding what learning looks like build participants' knowledge of the content focus through viewing video clips of individual children and examining Clements and Sarama's learning trajectories (2014). Finally, discussions focused on the implications of the big ideas for teaching and learning, as well as viewing and reflecting on the lesson videos that highlight activities that support children's foundational mathematics skills, are designed to support participants in their own classroom practice.

In addition to building teachers' knowledge and understanding of foundational early mathematics, each session provides early childhood educators with intentional teaching strategies and activities that support mathematics learning in young children that teachers can immediately take back and put into practice in their classrooms. The strategies and activities selected are based on the work of experts in the field of early mathematics including Clements and Sarama (2014) and the Erikson Institute Early Math Collaborative (2014). The participants have the opportunity during the professional development sessions to participate as learners in many of the activities that are suggested for use in their classrooms. These strategies and activities help teachers bring the concepts outlined in the big ideas of early mathematics to life for their students.

The sessions also emphasize how the big ideas of early mathematics align to the goals that preschool teachers have for children's learning including the Iowa Early Learning Standards (Early Childhood Iowa, 2012) and the objectives for development and learning in the Creative Curriculum for Preschool and Teaching Strategies GOLD assessment (Heroman et al., 2010). The sessions highlight how infusing the big ideas of early mathematics into early childhood programs meets standards for program quality presented in the NAEYC Early Childhood Program Standards (NAEYC, 2015) and the Iowa Quality Preschool Program Standards (Iowa Department of Education, 2006). Emphasizing the alignment of the big ideas to these standards helps support preschool teachers to see how the big ideas link to the expectations for children as well as program standards that are already in place.

Course Design

This professional development course, *Early Mathematics in Early Childhood Classrooms: An Exploration of the Big Ideas* includes nine, 4-hour training sessions. Session topics are listed below with a brief description.

- *Session 1: Setting the Stage for Our Exploration of the Big Ideas of Early Mathematics and Sorting It All Out:* This session focuses on setting the stage for the exploration of early mathematics that will take place in this course by outlining the reasons for digging deeper into early mathematics, the content that will be included, and how the sessions will be designed. This session also examines the first of the big ideas of mathematics, sets and sorting.
- *Session 2: Making Sense of Number Sense and Where's the Math in Blocks (Part 1)*: This session explores the big ideas of number sense, the power of blocks in early childhood classrooms, and the stages of block play.
- *Session 3: More than Just 1, 2, 3 and Math in Routines*: This session builds understanding of the big ideas of counting and how routines in the classroom support mathematical learning.
- *Session 4: The Story of Operations and Math in Games*: This session focuses on the big ideas of number operations and how games can be used to build children's understanding of mathematical content.

- *Session 5: Recognizing Repetition and Regularity and Where's the Math in Blocks (Part 2):* This session examines the big ideas of patterns and continues to build on the use of blocks in early childhood classrooms to support mathematical learning that began in Session 2 by examining relationships that can be brought out in block play.
- *Session 6: What Kind of "Big" is It? and Asking Questions and Finding Answers*: This session builds understanding of the big ideas of mathematics related to measurement and data analysis.
- *Session 7: Mapping the World Around Us and Where's the Math in Blocks (Part 3)*: This session explores the big ideas of spatial relationships and concludes the discussion on blocks in early childhood settings focusing on the mathematical thinking that children engage in when constructing with unit blocks.
- *Session 8: The Shape of Things and Good Math in Good Books*: This session focuses on the big ideas of shape and the use of children's books to engage children in mathematical learning.
- *Session 9: Big Connections with the Big Ideas*: This session explores the connections between the big ideas of early mathematics and how mindset impacts learning for both educators and children.

Course Materials

The following materials are needed throughout the sessions:

- Flip chart paper and markers
- Pens or pencils
- Timer

- Post-It notes
- Highlighters

Session Narratives

The learning objectives for each session are stated at the beginning of the narrative description of the session. The materials and participant handouts needed are also provided at the beginning of each session. A table outlining materials needed throughout the course is located in Appendix A. The presentation slides and participant handouts, organized by session, are provided in the appendices (see Appendix B-J). Within the narrative, topics in each session are capitalized, underlined, and in bold. Activities in each session are titled and in bold.

Session 1: Setting the Stage and Sorting It All Out

Learning Objectives

As a result of attending this session, participants will be able to:

1. Describe the importance of early mathematics and how this course supports teachers to develop children's foundational mathematics skills.
2. Summarize the big ideas of sets and sorting, how children's understanding of sets and sorting develops, and ideas for exploring sets and sorting in the classroom.

Materials

- Collections of sorting materials (at least one for each table)

Participant Handouts (see Appendix B)

- "Adult Perceptions of Their Abilities in Math" article (Worthington & Jones, 2007)

- *Early Childhood Mathematics: Promoting Good Beginnings* joint position statement (NAEYC & NCTM, 2002)
- Note-Catcher handout
- Note-Catcher Video Analysis handout
- Reflection handout

WELCOME AND INTRODUCTIONS

Due to the collaborative nature of these sessions, it is important for participants to develop trusting relationships with each other. To support this, the first session opens with an introductory activity in which the presenter and each participant shares their names, workplaces, the reason they are taking this course, and one practice currently used in their classrooms that supports children's foundational mathematics learning.

ADULT PERCEPTIONS OF MATHEMATICS

Activity – Read and Reflect

To encourage participants to reflect on their attitudes regarding mathematics, the participants read "Adult Perceptions of Their Abilities in Math" (Worthington & Jones, 2007). After reading the article, participants arrange themselves along a continuum between Mary, the teacher in the article who struggled with math through school and as an adult, and Suzanne, the teacher in the article who enjoyed math and made connections to real-life experiences. In groups of 3-4, participants share their stories within their groups and discuss how perceptions of math abilities affect classroom decisions and instruction. The group reconvenes to debrief the experience. One person from each group shares their story and groups share comments or themes that emerged during their discussions. Participants first reflect individually on what they want children's memories of math experiences to be like and the dreams they have for the children in their classrooms related to mathematical skills and abilities. Participants are asked to share some of their ideas to be recorded on chart paper to be referred back to over the course of the training sessions.

EARLY MATH AND EARLY MATH INSTRUCTION MATTERS

Duncan et al. (2007) highlighted the importance of supporting learning in the area of mathematics. In a meta-analysis of data sets from several longitudinal studies, children's mathematics knowledge and skills were found to be predictive of later achievement, not only in the area of mathematics, but also in the area of literacy. The National Research Council (2001) highlighted differences in reading and mathematics that stressed the importance of school-based instruction saying,

> School-based instruction may play a larger role in most children's mathematical experience than it does in their reading experience. If so, the consequences of good or poor mathematics instruction may have an even greater effect on children's proficiency than is the case with reading. (pp. 19-20)

Activity – Text Rendering Protocol – Joint Position Statement

Throughout this course, participants read and reflect on articles related to the session topics. To support participants, a text rendering protocol is used to provide a consistent framework for processing the articles (National School Reform Faculty, n.d.). The purpose of the experience is to collaboratively construct meaning, clarify, and expand thinking about the article. Each table will have someone designated as the facilitator whose job it is to guide the process and keep everyone on track and a recorder who will keep track of the phrases and words shared. To begin, each participant reads the article and notes the sentence, the phrase, and the word that he/she thinks is particularly important.

Three rounds of sharing are done in table groups. In the first round of sharing, each person shares a sentence that he/she believes is significant. In the second round, each person shares a phrase that he/she believes is significant. In the third round, each person shares the word that he/she believes is significant. The table groups discuss the ideas and themes that emerged during sharing and how the article confirms and/or challenges their own practices. As a large group, the presenter records the words that emerged and the whole group discusses new insights taken from the article. The first article participants will use this protocol for is the *NAEYC/NCTM Joint Position Statement* (NAEYC/NCTM, 2002, pp. 1-3). The remainder of the joint position statement will be read and processed as table groups. Each table reads one section of the recommendations sections and summarizes the key ideas, which are then shared with the entire group.

OVERVIEW OF COURSE DESIGN AND CONTENT

The sessions in this course are designed to address three areas that impact teaching and learning of mathematics in early child classrooms: teacher attitudes and beliefs, teacher knowledge and skills, and classroom practices (Chen & McCray, 2012). The activities are designed to promote a more positive disposition to the mathematics that is all around and to engage the participants in constructing their own understanding of the targeted Big Idea. The presentation and discussion of the Big Ideas, observation and discussion regarding what learning looks like, including video clips of individual children and the developmental trajectories (Clements & Sarama, 2014), are designed to build participants' knowledge of the content focus. Finally, discussion focused on the implications of the big ideas for teaching; and viewing and reflection on the lesson videos are designed to support participants in their own classroom practice.

The content for each session focuses on the big ideas of early mathematics (Clements & Sarama, 2014; Erikson Institute Early Math Collaborative, 2014). Clements and Sarama (2014) described big ideas of mathematics as, "clusters of concepts and skills that are mathematically central and coherent, consistent with children's thinking, and generative of future learning" (p. 3). The Erikson Institute Early Math Collaborative (2014) address 26 big ideas within 9 content topic areas. The big ideas are reflective of the Common Core State Standards and the work of NCTM, build upon each other, and compliment current practice (Clements & Sarama, 2014; Erikson Institute Early Math Collaborative, 2014). The Big Ideas are intended as organizers for adults. They were developed to add to adult knowledge and help focus teaching observation of these skills in children (Erikson Institute Early Math Collaborative, 2014).

Another important element of the sessions is the use of the strategic teaching practices highlighted by Erikson Institute Early Math Collaborative (2014) that support children's development of foundational mathematics knowledge and skills.

- "Mathematize the world around us" (p. 7). This practice involves looking at the world through a math lens and pointing out the math in everyday activities to children.
- "Make mathematics more than the manipulatives" (p. 7). Children's learning is not guaranteed by simply having math materials in the environment. Teachers must intentionally plan activities that help children make mathematical connections between concrete experiences, symbols, pictures, and language.
- "Recognize receptive understanding" (p. 8). In the same way that children are able to understand words they hear before they can actually say them

expressively, children are able to demonstrate their mathematical thinking and understanding nonverbally through gestures and actions. It is important that teachers are in tune to these expressions of receptive understanding.

- "Get mathematics into children's eye, ears, hands, and feet" (p. 9). Young children's learning hinges on their active engagement. Multimodal and multisensory opportunities are critical in deepening children's learning.
- "Scaffold children to construct their own understanding" (p. 9). The Erikson Institute Early Math Collaborative (2014) stated, "Our role as teachers is to guide them – to frame and orchestrate problem situations..." (p. 9). Teachers in early childhood classrooms design environments and experiences in which children question, problem solve, and discuss ideas.

Finally, each session connects the big ideas of mathematics to the learning standards, curriculum goals and objectives, and program standards that guide the work of early childhood teachers. As each content topic area is explored, participants see how the big ideas are representative of the mathematics standards in the Iowa Early Learning Standards (Early Childhood Iowa, 2012), the objectives for development and learning in the Creative Curriculum for Preschool and the Teaching Strategies GOLD assessment for the area of mathematics (Heroman et al., 2010), as well as several of the criteria in the NAEYC Early Childhood Program Standards and Accreditation Criteria (NAEYC, 2015) and the Iowa Quality Preschool Program Standards (Iowa Department of Education, 2006) curriculum standard.

SETS AND SORTING

Activity – Same and Different

The participants form a circle standing next to someone they don't know well. Introduce themselves to their neighbors and compare shoes with the person directly on the left and right. With the person on the left decide how the shoes are the same and with the person on the right decide how the shoes are different. Everyone in the circle introduces the person on the left and shares how their shoes are the same and introduces the person on the right and shares how their shoes are different. The participants are encouraged to not repeat similarities and differences that have already been shared. Once everyone has had an opportunity to share, the group reflects on the experience and begins a discussion of attributes.

Activity – Sorting Stuff & Sorting Again

Each table is provided with a collection of materials to sort. Some of the collections are made up of natural materials (rocks, shells, etc.) and some are sets of commercial manipulatives. Once the tables have completed the first sort, table groups share how they sorted their items and then are asked to sort them again in a different way. When the groups have completed the second sort, volunteers share how they sorted their items. This activity leads to a discussion on different ways that collections can be sorted including color, shape, material type, use, size, texture, etc.

Activity - Shoe Sort

The presenter poses a question to the group saying, "What kinds of shoes are we wearing today? How could we find out?" The participants gather in a circle and each person puts one shoe in the middle of the circle. A volunteer sorts the shoes into two groups using a binary sort. A binary sorting involves separating a collection into two sets, one made up of items possessing a certain attribute and one made up of items that do not have that attribute (Erikson Institute Early Math Collaborative, 2014). The rest of the participants try to figure out the rule for the sort. Additional participants are invited take turns sorting the shoes and rest of group determines the rule for the sort.

BIG IDEAS OF SETS AND SORTING

After the participants have experienced sets and sorting, the presenter poses the question, "What is mathematical about sorting?" This discussion leads to the presentation and discussion of three big ideas of sets and sorting:

- Attributes can be used to sort collections into sets.
- The same collection can be sorted in different ways.
- Sets can be compared and ordered (Erikson Institute Early Math Collaborative, 2014, p. 13).

Through discussion, the group processes how the preceding activities incorporated the big ideas of sets and sorting.

Activity – Video Analysis: Focus on the Child

Participants watch and reflect on video clips showing children presented with tasks regarding sets and sorting. The video clips, developed by the Erikson Institute Early Math Collaborative, are from one-on-one interviews with individual children. The interviews were designed to elicit evidence of children's mathematical thinking. They are not intended as teaching episodes or formal assessments. While watching, participants consider the following questions to make connections to the big ideas.

- What Big Ideas about sets and sorting do these children seem to understand?

- What Big Ideas about sets and sorting do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring sets and sorting might a teacher provide for these children to encourage them to develop their understanding further?

DEVELOPMENT OF CHILDREN'S THINKING ABOUT SETS AND SORTING

The discussion of the development of children's thinking around sets and sorting is based on the work of Clements and Sarama (2014) who presented classification as a mathematical process that children instinctively engage in even as infants in order to make sense of their world. The table below outlines the developmental progression of children's understanding of classification they identified (Clements & Sarama, 2014, p. 233).

Age	Developmental Progression
2 weeks	Distinguishes between objects they suck and those they do not.
2 years	Forms sets with objects that are similar.
3 years	Follows verbal rules for sorting.
4 years	Sorts objects according to a given attribute and form categories. May switch attributes during open sorting.
5 -6 years	Independently sorts by a single attribute and re-classifies by different attributes.

EXPLORING SETS AND SORTING IN THE CLASSROOM

Because of the connection to other mathematic concepts and the influence on mathematical thinking, understanding sets is foundational for young children. Early childhood environments offer plentiful natural opportunities for children to explore sets, but the teacher's role is critical in highlighting the mathematics and scaffolding children's learning around the topic. Many preschool classrooms have a multitude of commercially produced materials that can be used for sorting. It is, however, important for teachers to examine the natural materials they have available for children to sort. Not only should the types of materials be considered carefully, but also the factors that make sorting task more or less difficult for children. This includes the number of items, the number of attributes, the types of attributes, and the variances in a single attribute (Erikson Institute Early Math Collaborative, 2014).

Activity – Video Analysis: Focus on the Lesson

Participants watch and reflect on a video clip of the "People Sort" lesson in which the teacher engages her class in sorting and resorting each other to examine various attributes after reading the book *Five Creatures* (Jenkins, 2001, 2012). While viewing, participants consider the following questions to examine three elements of the lesson: the children, the teacher, and the activity itself.

About the Children

- What Big Ideas about sets and sorting do these children seem to understand?
- What Big Ideas about sets and sorting do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?

About the Teacher

- What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    - About materials to use?
    - About questions to ask?
    - About space arrangement?
- How does the teacher scaffold the children's thinking and explaining?

About the Activity

- How does the teacher tie the activity to the book *Five Creatures*?
- What modifications might you make if you were doing this activity in your classroom?
- What opportunities for exploring sets and sorting might a teacher provide for these children to encourage them to develop their understanding further?

MAKING CONNECTIONS TO THE BIG IDEAS OF SETS

To conclude the session, connections are drawn between the big ideas of sets and sorting and the learning standards, curriculum objectives and goals, and program standards that guide the work of preschool teachers in Iowa. The following table outlines the big ideas of mathematics for sets and sorting and the alignment to the various standards.

Big Ideas about Sets (Erikson Institute Early Math Collaborative, 2014)	• Attributes can be used to sort collections into sets; • The same collection can be sorted in different ways; and • Sets can be compared and ordered (p. 13).
Iowa Early Learning Standards (Early Childhood Iowa, 2012)	Standard: • Children understand comparisons and measurement. Benchmarks: • Children sort, classify, and put objects in series, using a variety of properties. • Children make comparisons between several objects based on one or more attributes (p. 132).
Objectives for Development & Learning (Heroman et al., 2010)	Objective: • Uses classification skills (p. 72)
NAEYC Early Childhood Program Standards (NAEYC, 2015) & Iowa Quality Preschool Program Standards (IQPPS) (Iowa Department of Education, 2006)	Standard: • Curriculum Criterion: • Children are provided varied opportunities and materials to categorize by one or two attributes, such as shape, size, and color (NAEYC 2.F.03, p. 17/IQPPS 2.24, p. 29).

Activity – Pause & Reflect

Before leaving the session, participants are asked to pause and reflect on their learning and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing, and/or stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom.

Session 2: Making Sense of Number Sense and Where's the Math in Blocks (Part 1)

Learning Objectives

As a result of attending this session, participants will be able to:

1. Summarize the big ideas of number sense, how children's understanding of number sense develops, and ideas for exploring numbers in the classroom.
2. Discuss the value of block play in early childhood classrooms and describe the stages of block play.

Materials

- Dot cards
- *Anno's Counting Book* Big Book (Anno, 1992)
- *Anno's Counting Book* (Anno, 1975/1977)
- Unit blocks

Participant Handouts (see Appendix C)

- "5" page layout from *Anno's Counting Book* (1975/1977)

- Note-Catcher handout
- Note-Catcher Video Analysis handout
- Learning Trajectory for Recognition of Number and Subitizing handout
- "Building Bridges to Understanding in a Preschool Classroom: A Morning in the Block Center" article (Christianson & James, 2015)
- Reflection handout

MAKING SENSE OF NUMBER SENSE

Activity – Number Soul Mates

As the participants enter the room, each is given an index card with a quantity of dots on it. In this activity, each participant finds the person whose card contains the same quantity of dots. Once everyone finds their number soul mate the pair takes turns asking each other what their favorite number is and why. Each pair introduces their partner and his/her favorite number.

Activity – *Anno's Counting Book*

The participants are introduced to *Anno's Counting Book* (Anno, 1992). The presenter shows the first several two-page layouts in the book and asks participants to share things that they notice about number on each page. On the page layout displaying five, the participants are asked to make a list with their tablemates of all the ways that five is represented on this page. Participants are given a handout with the page layout on it to examine more closely. After several minutes, each table group shares out one way they found five represented and the presenter records the list on chart paper. The table groups continue sharing in round-robin fashion until the lists are exhausted.

USE OF NUMBERS

While numbers are part of a patterned, infinite system that is used everyday, numbers can mean different things in different situations. The four uses of numbers include nominal or categorical numbers, referential numbers, cardinal numbers, and ordinal numbers. Nominal or categorical numbers are used as identifiers or names. Examples include room numbers, phone number, social security number, zip codes, or numbers on sports jerseys. This use of number is not inherently mathematical. Referential numbers are used as a reference point. Examples include time and temperature. This use is also not inherently mathematical. Both of these uses of number can be ordered but cannot be used in other ways in mathematical thinking or operations. The other two types of numbers, however, are essential for mathematical thinking. Cardinal numbers provide the answer to the questions, "How many?" and "How much?" Understanding of cardinal numbers is essential to meaningful counting and number operations. Ordinal numbers refer to position in a sequence and allow us to compare quantitative attributes. Children confuse cardinal and ordinal numbers, which is normal in their developmental process. It is important that educators clarify children's misconceptions and provide meaningful examples that help children develop a strong foundation in cardinal numbers.

Activity – What Can You See Quickly

Participants are shown several slides with small quantities of dots in varied arrangements and are asked to say how many dots are represented on each slide. This activity involves perceptual subitizing, quickly perceiving and naming "how many" objects are in a small collection. The word subitize comes from the Latin word "subito" which means, "to arrive suddenly" (Clements & Sarama, 2014). Clements and Sarama (2014) describe perceptual subitizing as the ability to "just see" how many objects in a small collection. Perceptual subitizing occurs in small sets of three or less objects.

Participants are shown several more slides with larger quantities of dots in varied arrangements, some of which are unorganized and others more structured. Participants are asked to say how many dots are represented on each slide. This part of the activity involves conceptual subitizing, or seeing the parts and putting together the whole number. Like perceptual subitizing, it happens quickly and often does not involve conscious thought (Clements & Sarama, 2014). Clements and Sarama (2014) emphasized subitizing as a foundational skill that very young children should develop but it does not develop on its own. In many classrooms there is not enough emphasis on subitizing. Educators must ensure they are encouraging children's subitizing skills rather than discounting them (Clements & Sarama, 2014).

NUMEROSITY

The presenter engages the group in a discussion regarding the idea of quantity as an attribute of a set comparing a set of three elephants to a set of three mice. This relates back to the first session in which the idea that a collection can have many attributes. This discussion is an introduction to the concept of numerosity, or the quantity of things in a set. Erikson Institute's Early Math Collaborative (2014) describes numerosity as the "three-ness" of 3. This concept is separate from number words and written symbols because those vary between languages but numerosity does not. It is important to remember that as children develop an understanding of numerosity, they need to be presented with many and varied experiences in which number words and numerals are used within a context that is meaningful and are used to describe something (The Early Math Collaborative, 2014).

BIG IDEAS OF NUMBER SENSE

After the participants have experienced and discussed subitizing and numerosity, they review how these activities relate to the three big ideas of number sense:

- Numbers are used in many ways, some more mathematical than others;
- Quantity is an attribute of a set of objects and we use numbers to name specific quantities; and
- The quantity of a small collection can be intuitively perceived without counting (Erikson Institute Early Math Collaborative, 2014, p. 29).

Activity – Video Analysis: Focus on the Child

Participants watch and reflect on video clips showing children presented with tasks involving number sense. While watching, participants consider the following questions to make connections to the big ideas.

- What big ideas about number sense do these children seem to understand?
- What big ideas about number sense do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring sets and sorting might a teacher provide for these children to encourage them to develop their understanding further?

DEVELOPMENT OF CHILDREN'S THINKING ABOUT NUMBER SENSE

Clements and Sarama's (2014) learning trajectory for the recognition of number and subitizing serves as the basis for the discussion of the development of children's thinking around number. The table below outlines the developmental progression of children's understanding of number identified in the related learning trajectory (Clements & Sarama, 2014, pp. 17-19).

Age	Developmental Progression
0-1 years	Does not have intentional knowledge of number.
1-2 years	Names groups of one to two.
3 years	Nonverbally makes a small collection with the same number.
4 years	Instantly recognizes collections up to four briefly shown and verbally names the number of items.
5 years	Instantly recognizes collections up to five briefly shown and verbally names the number of items. Verbally labels all arrangements to about five when shown only briefly. Verbally labels most briefly shown arrangements to six, then up to 10.
6 years	Verbally labels structured arrangements up to 20, shown only briefly, using groups.

EXPLORING NUMBER IN THE CLASSROOM

As stated earlier, subitizing is a foundational skill that must be developed in young children. Therefore, it is important for preschool teachers to understand the implications of the big ideas of number sense in the classroom. Subitizing relies on visual patterns; however, not all arrangements of a number are equally easy to "see." Therefore, children should be exposed to a variety of number arrangements. Children need opportunities to label small sets with number, without enumerating. Teachers should expect children to subitize small sets and should avoid forcing them to "count to be sure." This practice discredits children's subitizing. While subitizing is different than counting, it supports counting by building an understanding of cardinality. Children need authentic reasons to subitize and count small sets. It is important to restate the last count word when counting to emphasize cardinality: "1, 2, 3, ... 3 cups." As children developing number sense, smaller numbers are easier for them to understand than larger. With infants and toddlers, the focus should be on "1" and "2" and the idea of "1 more" and "2 more." With preschool aged children, teachers should spend a lot of time exploring "3" and "4" and "5." To support learning in young children, particularly in the area of mathematics, it is important that the abstract become concrete. Fingers are great tools for helping children understand small numbers, then building to 5 and 10. Children need repeated exposure to amounts in order to associate number name and quantity (Erikson Institute Early Math Collaborative, 2014).

There are a number of ways in which teachers can support children's development of number sense through the use of models. One model, dot cards, was used in the opening activity of this session. Dot cards that display quantities in different arrangements can be created using dot stickers. Another model, number cards, combines dots or other representations of quantity with the written numeral. Dice serve as another model to support number sense. Ten-frames are another visual model for number sense. Ten is important because it is the keystone to our number system. But it is too difficult for preschool aged children to develop a firm understanding of it. Five-frames could also be used as a model to support visual number sense for younger children. Finally, counting frames, or "rekenreks," can be used as visual models to support children's development of number sense. The rekenrek is a tool that was created in the Netherlands by Adrian Treffers, a mathematics curriculum researcher. Directly translated, rekenrek means calculating frame or arithmetic rack and it is designed to support the natural mathematical development of children. The color of the beads changes after 5 beads so that children can subitize easier and relate quantities to the benchmarks of 5 and 10. The rekenrek supports even the youngest learners with the visual models they need to discover number relationships and develop automaticity.

Activity – Video Analysis: Focus on the Lesson

Participants watch and reflect on a video clip of the "Number Arrangements" lesson in which children create visual representations of different numbers. While viewing, participants consider the following questions to examine three elements of the lesson: the children, the teacher, and the activity itself.

About the Children

- What big ideas about number do these children seem to understand?
- What big ideas about number do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?

About the Teacher

- What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
  - About materials to use?
  - About questions to ask?
  - About space arrangement?
- How does the teacher scaffold the children's thinking and explaining?

About the Activity

- What does it do for their children's understanding to describe their arrangements using numbers?
- What modifications might you make if you were doing this activity in your classroom?

- What opportunities for exploring number might a teacher provide for these children to encourage them to develop their understanding further?

MAKING CONNECTIONS TO THE BIG IDEAS OF NUMBER SENSE

Connections are drawn between the big ideas of number sense and the learning standards, curriculum objectives and goals, and program standards that guide the work of preschool teachers in Iowa. The following table outlines the big ideas of mathematics for number sense and the alignment to the various standards.

Big Ideas about Number Sense (Erikson Institute Early Math Collaborative, 2014)	• Numbers are used in many ways, some more mathematical than others; • Quantity is an attribute of a set of objects and we use numbers to name specific quantities; and • The quantity of a small collection can be intuitively perceived without counting (p. 29).
Iowa Early Learning Standards (Early Childhood Iowa, 2012)	Standard: • Children understand counting, ways of representing numbers, and relationships between quantities and numbers. Benchmarks: • Children begin to recognize small quantities without counting them. • Children start recognizing and naming numbers (p. 121).

Objectives for Development & Learning (Heroman et al., 2010)	Objective:
	• Uses number concepts and operations.
	  • Quantifies
	  • Connects numerals with their quantities (pp. 107-108)

NAEYC Early Childhood Program Standards (NAEYC, 2015) & Iowa Quality Preschool Program Standards (IQPPS) (Iowa Department of Education, 2006)	Standard: • Curriculum Criterion: • Children are provided varied opportunities and materials to build understanding of numbers, number names, and their relationship to object quantities and to symbols. (NAEYC 2.F.03, p. 17/IQPPS 2.23, p. 29).

THE POWER OF BLOCKS

Research indicates that young children's block play helps build a stronger foundation for learning mathematics. A longitudinal study that began in 1984 tracked 37 preschoolers and kindergartners until 2001 when they were in high school and found that those who had more sophisticated block play early on got better math grades and standardized test scores in high school (Wolfgang, Stannard, & Jones, 2001). Early block play predicts conceptual understanding of geometry and mathematical equivalence in elementary school (Peterson & Levine, 2014) and supports spatial reasoning, which is linked to strengthening mathematical skills over time (Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2014).

Activity – Text Rendering Protocol – "Building Bridges" article

The purpose of the text rendering experience is to collaboratively construct meaning, clarify, and expand thinking about the article. Each participant reads the article, "Building Bridges to Understanding in a Preschool Classroom: A Morning in the Block Center" (Christianson & James, 2015), and notes the sentence, the phrase, and the word that he/she thinks is particularly important. Three rounds of sharing are done in table groups. In the first round of sharing, each person shares a sentence that he/she believes is significant. In the second round, each person shares a phrase that he/she believes is significant. In the third round, each person shares the word that he/she believes is significant. The table groups discuss the ideas and themes that emerged during sharing and how the article confirms and/or challenges their own practices. As a large group, the presenter records the words that emerged and the whole group discusses new insights taken from the article.

Activity – Building Challenge

Using the unit blocks provided, each table group builds a model of an important Chicago landmark, the Hancock Building. As participants build, different views of the Hancock Building are displayed on the screen. After several minutes of building, the group debriefs the mathematics they experienced during this building activity.

STAGES OF BLOCK BUILDING

All children progress through specific stages as they use blocks in play (Hirsh, 1996). In the first stage, discovery, children carry, move, touch, hold, pile, knock down, and drop the blocks. Little or no actual building happens during this stage. In the next stage, towers and roads, children stack blocks vertically or line them up in rows horizontally. There is a great deal of repetition in their building. It is in this stage that the first application of imagination occurs as props such as cars or trucks are used on the "roads." The next stage, bridges, is when children begin to experiment connecting two blocks with a space between them with a third block to make a bridge or a doorway. In this third stage, they explore balance and rotate blocks to use their widths and ends of the blocks. Children learn to bridge by trial and error as they begin to explore ideas about measurement and balance. The fourth stage, enclosures, is demonstrated when children can close up space between blocks with another block(s) to make walls, fences, rooms, cages. They often add figures, incorporating animals, people, signs, cars, etc. The fifth stage, patterns and symmetry, is characterized by more elaborate, decorative structures, using pattern, symmetry, and balance through deliberate efforts. Children at this stage sort and match blocks' shapes and sizes to find equivalences and often name their structures. Finally, in the sixth stage, representation building, children plan and build elaborate structures, revising their plans as they work. When building together, they often assign each other builder roles. They use a variety of materials to achieve desired effects. They spend much time sorting, matching, and arranging. In this stage, structures often symbolize actual buildings or places children know. Children at this stage often want to build and play with a structure over several days. While there are ages at which these stages are often seen, it varies depending on children's experience with blocks.

Activity – Reflecting on Children's Block Building Stage

Participants examine several photographs of children engaged in block building. Based on the photos, participants identify the block building stage the children are exhibiting describe the evidence they see that demonstrates that stage.

Activity – Pause & Reflect

Before leaving the session, participants are asked to pause and reflect on their learning and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing and stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom.

Session 3: More than Just 1, 2, 3 & Math in Routines

Learning Objectives

As a result of attending this session, participants will be able to:

1. Summarize the big ideas of counting, how children's understanding of counting develops, and ideas for exploring counting in the classroom.
2. Discuss how to support mathematics within classroom routines.

Materials

- Counting jar (one for each table) containing a small number of items
- Rekenrek attendance chart
- Popsicle sticks

Participant Handouts (see Appendix D)

- Note-Catcher handout
- Note-Catcher Video Analysis handout
- Learning Trajectory for Counting handout
- "Calendar Time for Young Children: Good Intentions Gone Awry" article (Beneke, Ostrosky, & Katz, 2008)
- Reflection handout

COUNTING: MORE THAN JUST 1, 2, 3

Activity – Counting Jars

Each table is provided with a counting jar that holds a specified number of items for them to count without touching the items. Participants write a prediction for the number of items and then spill out the items to count them.

TWO TYPES OF COUNTING

The task of counting can be broken out into two types: rote counting and rational counting. Rote counting is the recitation of number names in order from memory. Rational counting involves matching each number name in order to an object in a collection. The functionality of rote counting is limited; it has a role in learning to count. However, rational counting is foundational in children's understanding of working with numbers.

Activity – Discussion – Why do we count?

The reason we count is to find out "how many" or "how much." We do this so often that we don't even realize that we are engaged in mathematical thinking. It is so simple, yet complex. Understanding "how many" is closely related to developing number sense in several ways. Counting is the cardinal use of numbers, which were discussed in the second session. Counting is closely linked to children's ability to subitize. Counting is also tied to the concept of numerosity, or the "three-ness" of three, which has little to do with the actual objects being counted. Number names are used to temporarily label items as they are counted no matter what they are. Finally, as was discussed with number sense, counting experiences must be meaningful for young children in order for them to make sense of the concept.

PRINCIPLES OF COUNTING

There are four principles, or rules, of counting that preschool teachers need to be familiar with in order to recognize where children are in their development of thinking about counting: stable order, one-to-one correspondence, order irrelevance, and cardinality.

The stable order principle is the idea that counting words have to be said in the same order every time. The order in which numbers are said when counting is fixed. Three always comes after two and before four. While counting does involve memorization, there are mathematical structures and patterns within this principle. Each number is always one more than the number before it and one less than the number after. The number sequence includes patterns that help us understand the sequence and place value. The number system is based on a system of 10s. The numerals 0-9 are used and there are patterns to how numbers are represented. The number names for the teen numbers, however, do not follow these patterns, which is the reason they cause confusion for many children as they are learning them.

The next principle, one-to-one correspondence is the concept that one number is named for each object. While this is simple for adults to understand, it does take time for children to master because it involves the coordination of number words with the physical movement of a finger and eyes along a line of objects, saying one number word to one object until the items run out. There are three common errors that often observed as children are learning this principle. In one type of error, children have the correct correspondence but incorrect sequences (e.g., a child points to each item in collection but says, "1, 2, 3, 5, 6, 4, 10, 8"). This error occurs when children do not have an understanding of stable order. Another error involves the correct sequence but incorrect correspondence. Children demonstrate this error by saying the numbers in the right order but "double tag" some objects because they are counting too fast. The third error also involves correct sequence but incorrect correspondence. Again, children say the numbers in the right order but skip over tagging some items because they are pointing too fast.

The third principle, order irrelevance, is that no matter in what order the items in a collection are counted, the result is the same. This principle builds on the rule of stable order and generalizes the idea behind one-to-one correspondence. Children who grasp what it means to count understand that it doesn't matter how a group of objects is arranged; arrangement does not affect the total number. The key is that each object is only counted once. Counting scattered groups of objects in everyday situations requires a system to mark the items that have been counted and which have not. Some common strategies include pushing items counted into piles, marking next to items counted, putting items from a large collection into clusters of 2s, 5s, or 10s to use skip counting.

The final rule of counting is the cardinality principle: the last number name said when a count is finished is what tells how many items in the set. Children demonstrate the cardinality principle in the following ways: when asked, "How many altogether?" naming the last number counted without recounting; counting out a given quantity; counting on or counting back from a given quantity rather than counting all; and knowing a quantity remains the same even when items are arranged differently (Erikson Institute Early Math Collaborative, 2014).

BIG IDEAS OF COUNTING

After the participants have experienced and discussed counting, they review how these activities relate to the two big ideas of counting:

- Counting can be used to find out "how many" in a collection; and
- Counting has rules that apply to any collection (Erikson Institute Early Math Collaborative, 2014, p. 47).

Activity – Video Analysis: Focus on the Child

Participants watch and reflect on video clips showing children presented with tasks involving counting. While watching, participants consider the following questions to make connections to the big ideas.

- What big ideas about counting do these children seem to understand?
- What big ideas about counting do these children seem to be learning?

- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring counting might a teacher provide for these children to encourage them to develop their understanding further?

DEVELOPMENT OF CHILDREN'S THINKING ABOUT COUNTING

Clements and Sarama's (2014) learning trajectory for counting serves as the basis for the discussion of the development of children's thinking around counting. The table below outlines the developmental progression of children's understanding of counting in the related learning trajectory (Clements & Sarama, 2014, pp. 36-43).

Age	Developmental Progression
2 years	Verbally counts to 5. Uses many-to-one correspondence or overly rigid one-to-one correspondence.
3 years	Verbally counts to 10. May have some errors in one-to-one correspondence. Keeps one-to-one correspondence for a small number of objects. When asked "how many?" will recount the set.
4 years	Counts small sets of object in a line. Answers "how many" questions with the last number counted. Beginning to understand number just before and just after another number. Developing verbal counting to 20. Produces a set of a certain number of objects.
5 years	Counts out objects to 10. Understands cardinality principle. Uses strategies to keep track of objects counted.
6 years	Counts verbally and with objects from numbers other than 1 (but does not yet keep track of the number of counts). Immediately determines numbers just after or just before. Skip counts by 10s up to 100 or beyond with understanding. Counts to 100. Makes decade transitions (e.g., from 29 to 30) starting at any number. Keeps track of a few counting acts, by only using numerical pattern (spatial, auditory or rhythmic). Counts by fives or twos with understanding. Counts mental images of hidden objects.

EXPLORING COUNTING IN THE CLASSROOM

Teachers need to support children to develop rational counting skills through authentic experiences. One way in which this can be done is creating contexts in which children need to know "how many?" As discussed in the session on number sense, children need to be provided many opportunities for counting that are focused on small numbers. It is also important to include movement or other cues to help children develop understanding of number system.

The routines within the preschool day provide many opportunities to practice counting including attendance, snack, and large group times. The use of routines to support mathematical learning will be expanded on later in this session.

The following table outlines activities suggested by the Erikson Institute Early Math Collaborative (2014) that build children's understanding of the principles of counting.

Stable Order	One-to-One Correspondence	Order Irrelevance	Cardinality
• Counting songs & movement games • Counting up and back from a number • Post and reference a 1-10 number line with dots and numerals	• Daily routines • Music and movement • Board games with paths to move along spaces	• Start counts for a fixed set in different order • Use think-alouds and model using a system – lining up, clustering, counting by 2s, 5s, or 10s	• Label the cardinal value of a set after counting (e.g., 1, 2, 3, 4… 4 books) • Count out a specified number

The number system follows a predictable pattern. Once the structure is understood, an individual can count accurately to any number. It is a base-10 system: when the next 10 is reached, the sequence begins again. When children understand this concept, it makes large numbers much more manageable.

Activity – Video Analysis: Focus on the Lesson

Participants watch and reflect on a video clip of the "Movement Counts" lesson in which children roll a die and act out movements depicted in *From Head to Toe* (Carle, 1997) to connect counting words, visual number arrangements, and actions. While viewing, participants consider the following questions to examine three elements of the lesson: the children, the teacher, and the activity itself.

About the Children

- What big ideas about counting do these children seem to understand?
- What big ideas about counting do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?

About the Teacher

- What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    - About materials to use?
    - About questions to ask?
    - About space arrangement?
- How does the teacher scaffold the children's thinking and explaining?

About the Activity

- What does it do for the children's understanding to connect numbers with movement?
- What modifications might you make if you were doing this activity in your classroom?
- What opportunities for exploring counting might a teacher provide for these children to encourage them to develop their understanding further?

MAKING CONNECTIONS TO THE BIG IDEAS OF COUNTING

Connections are drawn between the big ideas of counting and the learning standards, curriculum objectives and goals, and program standards that guide the work of preschool teachers in Iowa. The following table outlines the big ideas of mathematics for counting and the alignment to the various standards.

Big Ideas about Counting (Erikson Institute Early Math Collaborative, 2014)	• Counting can be used to find out "how many" in a collection; and • Counting has rules that apply to any collection (p. 47).
Iowa Early Learning Standards (Early Childhood Iowa, 2012)	Standard: • Children understand counting, ways of representing numbers, and relationships between quantities and numbers. Benchmarks: • Children count to five. • Children count objects, pointing to each one correctly while counting (p. 121).

Objectives for Development & Learning (Heroman et al., 2010)	Objective:
	• Uses number concepts and operations.
	  • Counts
	  • Quantifies
	  • Connects numerals with their quantities (pp. 107-108)

NAEYC Early Childhood Program Standards (NAEYC, 2015) & Iowa Quality Preschool Program Standards (IQPPS) (Iowa Department of Education, 2006)	Standard: • Curriculum Criterion: • Children are provided varied opportunities and materials to build understanding of numbers, number names, and their relationship to object quantities and to symbols. (NAEYC 2.F.03, p. 17/IQPPS 2.23, p. 29).

MATH IN ROUTINES

Activity – Attendance Chart

As they come into the room, participants place a craft stick in a pocket on the chart for a later activity.

Activity – Video Analysis: Focus on Routines

Participants watch and reflect on video clips showing children presented with tasks related to routines. While watching, participants consider the following questions to make connections to the big ideas and the use of routines.

- What, specifically, do these children say or do that gives you evidence of their thinking?
- What big ideas does this routine reinforce for children?

Activity – Calendar Routine Turn and Talk

In pairs, participants discuss how they do or do not use the calendar as a part of the classroom routine.

Activity – Text Rendering Protocol – "Calendar Time for Young Children" article

The purpose of the experience is to collaboratively construct meaning, clarify, and expand thinking about the article. Each participant reads the article, "Calendar Time for Young Children: Good Intentions Gone Awry" (Beneke, Ostrosky, & Katz, 2008), and notes the sentence, the phrase, and the word that he/she thinks is particularly important. Three rounds of sharing are done in table groups. In the first round of sharing, each person shares a sentence that he/she believes is significant. In the second round, each person shares a phrase that he/she believes is significant. In the third round, each person shares the word that he/she believes is significant. The table groups discuss the ideas and themes that emerged during sharing and how the article confirms and/or challenges their own practices. As a large group, the presenter records the words that emerged and the whole group discusses new insights taken from the article.

Activity – Examples of Building Math into Routines

A linear calendar can be used in place of a traditional 7-column calendar. The linear calendar displays the numbers 1 through 28, 30 or 31 along a strip of paper. Upcoming events can be noted on the calendar in advance and special events that take place can be added as well. Each day a clip or some type of marker is used to indicate the date.

A rekenrek-styled attendance chart, similar to the one used in the attendance chart activity earlier in this session, can be used to take daily attendance. Students are encouraged to discover efficient ways to answer, "How many children are here today?" At the beginning of the year, they might use counting by ones as the strategy to respond to this question. As children explore the structure of the rekenrek counting rack tool, however, they begin to unitize and count by fives and tens. This daily routine gives students the opportunity to practice subitizing and gradually builds important numerical relationships at the core of number sense.

Nearly every early childhood classroom has some way of marking the number of days that school has been in session. Teachers might consider using a ten-frame for this purpose.

Activity – Video Analysis – Kindergarten Counting Jar Routine

Participants watch and reflect on a video clip of a counting routine in a kindergarten classroom. In the video, the teacher will introduce the routine and one boy shares his counting strategies with the class. While watching, participants consider the following questions to make reflect on the value of the use of routines to teach mathematical skills.

- What, specifically, do these children say or do that gives you evidence of their thinking?

- What big ideas does this routine reinforce for children?

Activity – Pause & Reflect

Before leaving the session, participants are asked to pause and reflect on their learning and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing and stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom.

Session 4: The Story of Operations & Math in Games

Learning Objectives

As a result of attending this session, participants will be able to:

1. Summarize the big ideas of number operations, how children's understanding of number operations develops, and ideas for exploring counting in the classroom.
2. Discuss how games support children's mathematical learning.

Materials

- Operations symbols to post including + - x /
- *The Gingerbread Man* (Mackinnon, 2007)
- Materials for games
    - Game of NIM - 15 pennies or counters
    - Achi - Achi game board, 4 counters of one color and 4 counters of another color

- Balloon Volleyball - Balloon, way of recording tally marks
- Itsy Bitsy Spider Game - Dot cube, egg carton without a lid, "spider" game piece such as plastic spider or spider ring
- Matching Cards - Set of cards with matching numbers represented in two different ways (dot arrangements and numerals, for example)

Participant Handouts (see Appendix E)

- Note-Catcher handout
- Note-Catcher Video Analysis handout
- Learning Trajectory for Addition and Subtraction handout
- Learning Trajectory for Comparing, Ordering, and Estimating handout
- Math Games handouts
- Reflection handout

THE STORY OF OPERATIONS

Activity – Operations Four Corners

Four arithmetic operation symbols are displayed on the walls in the four corners of the room. Participants are asked, "Which operations symbol most resonates with you?" and they move to that corner of the room and talk about the reasons for choosing that symbol with the others in the group. Each symbol group shares out to the large group the common themes from the discussion.

NUMBER OPERATIONS

The Erikson Institute Early Math Collaborative (2014) stressed that children's ability to make sense of number operations develops from a foundational understanding that every operation tells a story. Number operations help answer questions such as, "How many now?" "How many more?" "How many less?" and "Is it fair?" Number operations are grounded in mathematical problems that examine the relationships between numbers. Number operations examine changes in sets, comparisons between sets, as well as part and whole relationships. Two types of strategies can be used to address mathematical problems in number operations: direct modeling and counting strategies. Direct modeling strategies involve using concrete objects to model the problem and counting to find the solution. As children become more experienced and comfortable with the number system, they begin to use counting strategies, numbers in place of physical representations in a problem (Erikson Institute Early Math Collaborative, 2014).

Activity – Story Acting – The Gingerbread Man

The presenter introduces the book to the group. It is the basis for a retelling of the story. While reading the book, the presenter pauses and chooses individuals from the group to play the role of the characters in the story. Throughout the reading, the presenter pauses and asks the audience "How many now?"

BIG IDEAS OF NUMBER OPERATIONS

After the participants have discussed and experienced number operations, they review how these activities relate to the two big ideas of number operations:

- Sets can be changed by adding items (joining) or by taking some away (separating);
- Sets can be compared using the attribute of numerosity, and ordered by more than, less than, and equal to; and

- A quantity (whole) can be decomposed into equal or unequal parts; the parts can be composed to form the whole (Erikson Institute Early Math Collaborative, 2014, p. 65).

Activity – Video Analysis: Focus on the Child

Participants watch and reflect on video clips showing children presented with tasks involving number operations. While watching, participants consider the following questions to make connections to the big ideas.

- What big ideas about number operations do these children seem to understand?
- What big ideas about number operations do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring number operations might a teacher provide for these children to encourage them to develop their understanding further?

DEVELOPMENT OF CHILDREN'S THINKING ABOUT NUMBER OPERATIONS

Clements and Sarama's (2014) learning trajectory for addition and subtraction as well as the learning trajectory for comparing, ordering, and estimating serve as the basis for discussion of the development of children's thinking around number operations. The table below outlines the developmental progression of children's understanding identified in the two related learning trajectories (Clements & Sarama, 2014, pp. 57-63, pp. 84-88).

Age	Developmental Progression
2 years	Uses words to include "more", "less" or "same." Implicitly sensitive to the relation of more than/less than involving very small numbers. Adds and subtracts very small groups nonverbally.
3 years	Compares collections of 1 to 4 items verbally or nonverbally ("just by looking") when the items are the same. May compare the smallest collections using number words "two" and "three", and "three" and others. Adds and

	subtracts very small groups nonverbally.
4 years	Matches small, equal collections, showing that they are the same number. Compares groups of 1 to 6 by matching.
4-5 years	Finds sums for joining ("You had 3 apples and get 3 more, how many do you have in all?") and part-part whole ("There are 6 girls and 5 boys on the playground, how many children were there in all?") problems by *direct modeling, counting all, with objects.* Solves take-away problems by separating with objects. Adds on objects to "make one number into another," without needing to count from "one." Finds the missing addend (5 + _ = 7) by adding on objects. Compares by matching in simple situations.
5 years	Compares with counting, even when larger collection's objects are smaller. Later, figures out how many more or less. Identifies and uses ordinal numbers from "first" to "tenth."
5-6 years	Finds sums for joining ("You had 8 apples and get 3 more...) and part-part-whole (6 girls and 5 boys...) problems with finger patterns and/or by counting on.
6 years	Uses internal images and knowledge of number relationships to determine relative size and position. Orders numbers in a collection (small numbers first). Has initial part-whole understanding. Solves all previous problems using flexible strategies.

EXPLORING COUNTING IN THE CLASSROOM

It is not necessary to focus on arithmetic and the symbols used for addition and subtraction with young children. Instead, they need experiences that focus on how the relationships between quantities work in real life situations. Focusing on the relationships builds the foundation children will need to understand a problem that is only represented in symbols as they progress through school. To do this, teachers must foster children's strategies for problem solving. This requires adult planning and action to actively engage all children in making sense of the problem situation and number relationships. Adults also need to support children's use of both direct modeling and counting strategies. The focus should be more on understanding rather than on just right answers. Exposing children to a range of problem solving strategies encourages flexibility in thinking through operations in whatever way works best. To do this, children need repeated opportunities to use concrete objects, draw pictures to show thinking, and explain and discuss possible solutions. Rather than telling children they are wrong or giving them the right answer, adults can pose questions such as, "Why do you think that?" or "Does anyone else have a comment or different idea?"

Activity – Video Analysis: Focus on the Lesson

Participants watch and reflect on a video clip of the "Mouse Collections" lesson in which the teacher uses *Mouse Count* (Walsh, 1991) to explore the concepts of "more" and "less". While viewing, participants consider the following questions to examine three elements of the lesson: the children, the teacher, and the activity itself.

<u>About the Children</u>

- What big ideas about number operations do these children seem to understand?
- What big ideas about number operations do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?

<u>About the Teacher</u>

- What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    - About materials to use?
    - About questions to ask?
    - About space arrangement?
- How does the teacher scaffold the children's thinking and explaining?

<u>About the Activity</u>

- How does the teacher tie the activity to the book *Mouse Count*?

- What modifications might you make if you were doing this activity in your classroom?
- What opportunities for exploring number operations might a teacher provide for these children to encourage them to develop their understanding further?

MAKING CONNECTIONS TO THE BIG IDEAS OF NUMBER OPERATIONS

Connections are drawn between the big ideas of number operations and the learning standards, curriculum objectives and goals, and program standards that guide the work of preschool teachers in Iowa. The following table outlines the big ideas of mathematics for number operations and how they align to the various standards.

Big Ideas about Number Operations (Erikson Institute Early Math Collaborative, 2014)	• Sets can be changes by adding items (joining) or by taking some away (separating); • Sets can be compared using the attribute of numerosity, and ordered by more than, less than, and equal to; and • A quantity (whole) can be decomposed into equal or unequal parts; the parts can be composed to form he whole (p. 65).
Iowa Early Learning Standards (Early Childhood Iowa, 2012)	Standard: • Children understand counting, ways of representing numbers, and relationships between quantities and numbers. Benchmark: • Children use language such as more or less to compare quantities (p. 121).
Objectives for Development and Learning (Heroman et al., 2010)	Objective: • Uses number concepts and operations. • Quantifies (p. 107)
NAEYC Early Childhood Program Standards (NAEYC, 2015) & Iowa Quality Preschool Program Standards (IQPPS) (Iowa Department of Education, 2006)	Standard: • Curriculum Criterion: • Children are provided varied opportunities and materials to build understanding of numbers, number names, and their relationship to object quantities and to symbols (NAEYC 2.F.03, p. 17/IQPPS 2.23, p. 29).

MATH IN GAMES

Activity – The Game of NIM

In pairs, participants play several rounds of The Game of NIM. The goal of this game is to reason quantitatively and abstractly in order to leave your opponent with the last counter. To play, 15 counters are spread out over the playing surface. The players take turns taking 1, 2, or 3 counters from the set. The player who takes the last counter loses the game. After playing several rounds, participants debrief the experience discussing why one player won over the other. This is a strategy game that promotes thinking, logical relationships, reasoning, problem solving, and patterns. Versions of this game can be found in many cultures.

WHAT ARE MATH GAMES

Oldfield (1991) outlined four components that define math games. First, a math game is an activity that involves a challenge. The challenge could be against a task or against one or more opponents. The challenge could also be a shared task that is addressed either by each participant or collaboratively as a group. Second, math games are governed by a set of rules and have a clear underlying structure. The third component defining math games is that the activity normally has a distinct ending point. Finally, to fit the definition of a math game, an activity must also have specific mathematical cognitive objectives.

The use of math games benefits motivation, access, and learning for children. In terms of motivation, math games offer meaningful situations for the application of mathematical skills. Children enjoy playing and choose to participate. Math games also offer an opportunity for children to build self-concept and develop positive attitudes towards mathematics, through reducing the fear of failure and error.

With regard to access, math games are accessible to all children as there are few language barriers and they build children's independence. Children can work independently of a teacher. Math games also build home and school connections when families are encouraged to play games at home as well as share games that are played at home with the classroom community. Math games also increase children's exposure to mathematical concepts. According to Davies (1995) math games allow children to operate at different levels of thinking and to learn from each other. They also promote children's logical thinking as they test intuitive ideas and problem solving strategies in games. Finally, math games offer teachers opportunities for assessment of learning in non-threatening situations.

Activity – Video Analysis: Tapatan Game

Participants watch and reflect on a video clip showing two children playing a round of Tapatan. The goal of the game is to get three counters of one color in a row. While watching, participants consider the following questions to make connections to the big ideas and the use of games.

- What, specifically, do these children say or do that gives you evidence of their thinking?
- What evidence of motivation or access do you see in this video?

Activity – Game Station Rotation

In table groups, participants will rotate through four stations to experience four different types of math games. Instructions and materials for each game are arranged at four tables around the room. Each group has approximately 5 minutes at each station. While at each station, participants try out the game, discuss what kind of mathematical thinking the game activates, and discuss what other games are similar. A piece of chart paper is displayed at each station for groups compile the ideas for similar games. The four games are:

- Achi is a strategy game similar to the Tapatan game seen in the video clip. To play, two players take turns placing one counter on a circle. If three counters of one color are in a row, that player calls out "Achi" and wins the round. If both players have played all their pieces and no one has won, then players take turns sliding a counter on a line to the empty place. When a player makes a line of three of his/her counters by sliding, that player calls out "Achi" and wins the round. The game can be played over and over again for many rounds so that both players will win and develop strategic thinking.
- Balloon volleyball is a gross motor game. The goal is to count using one-to-one correspondence to determine number of balloon taps and to compare number of taps for each round. Players sit on the floor divided into two groups. Have each group arrange themselves in rows like on a volleyball court. A balloon is tossed into the center of the two groups. Players try to keep the balloon up in the air by tapping it with their fingers while they remain sitting. Players count each balloon tap and make a tally mark for every tap. Players try to make as many taps as possible before the balloon touches the floor. Play at least three times. Compare the number of taps for each game to determine the best strategy for balloon volleyball.
- Itsy Bitsy Spider game is an example of a path game. The goal is to count spaces on a game board with one-to-one correspondence and to subitize small quantities

of dots. An egg carton is placed vertically so that it represents the waterspout. Players take turns rolling the dot cube, subitizing (or counting) the number of dots, and then moving their spiders that number of spaces—first up and then down the "spout." The players can decide together whether the spiders need to land exactly in the last cup. Play ends when both spiders complete the path.

- Memory is a matching card game that can be played with two players. For this version, two sets of cards with numbers represented in two different ways (e.g., dot arrangements and numerals). Place dot cards face down in one row and the numeral cards face down in another row. Children take turns turning over one dot card and one numeral card. They say the number name for each card. If the cards match, the player keeps the cards. If the cards do not match, they are turned facedown again. Players take turns until all the cards are matched. This game can also be adapted in several ways. It can be played with all cards face-up at first, and then with one set (dots or numerals) face-up and the other set face down. Some children may also benefit from matching numeral cards to numeral cards and/or dot cards to dot cards. The representations on the cards could also be changed. For example, five-frame or ten-frame cards could be used.

Activity – Pause & Reflect

Before leaving the session, participants are asked to pause and reflect on their learning and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing and stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom.

Session 5: Recognizing Repetition and Regularity & Where's the Math in Blocks (Part 2)

Learning Objectives

As a result of attending this session, participants will be able to:

1. Summarize the big ideas of pattern, how children's understanding of pattern develops, and ideas for exploring pattern in the classroom.
2. Discuss the relationships between blocks and how blocks can be used to support children's mathematical learning.

Materials

- A variety of materials for creating patterns
- Chart paper labeled with the following headings: children's artwork, blocks, songs and finger plays, instruments, outside, and books
- Unit blocks

Participant Handouts (see Appendix F)

- Note-Catcher handout
- Note-Catcher Video Analysis handout
- Learning Trajectory for Pattern handout
- Reflection handout

PATTERNS: RECOGNIZING REPETIION AND REGULARITY

Activity – Pattern Walk

Participants are given 5 minutes to walk around the building, either inside or outside and look for patterns in the environment. The group discusses the patterns they noticed on their pattern walk. When implemented in the classroom, this type of activity can be used to informally assess children's receptive understanding of pattern. It is also a way to mathematize the patterns that children might see every day and help them to notice these patterns.

PATTERNS AND STRUCTURE

A pattern is defined as, "Any predictable sequence found in physical and geometric situations as well as in numbers" (Erikson Institute Early Math Collaborative, 2014, p. 83). Patterns come in many forms: visual patterns, auditory patterns, movement patterns, temporal patterns, and numerical patterns. Structure is defined as, "The ways in which various elements in a pattern are organized and related" (Erikson Institute Early Math Collaborative, 2014, p. 83). Patterns and structure are more defining qualities of mathematics than they are topics of mathematics; they are the very foundation of mathematics. Mathematics begins to make sense because patterns allow us to generalize our understanding from one situation to another. Children need many opportunities to discover and talk about patterns in mathematics. These learning opportunities help children develop the positive attitudes toward mathematics and confidence that mathematics should make sense. Developing these attitudes and confidence toward mathematics helps children to become persistent problem solvers. Because children are naturally drawn to notice patterns, teachers can build on this and help them to become more precise and mathematical in their thinking.

Activity – Creating Patterns

Each participant creates a pattern with the materials provided at each table or other materials they can access. The presenter notes particular patterns that represent the different types of patterns to be highlighted during the discussion.

TYPES OF PATTERNS

There are four types of patterns: repeating patterns, growing patterns, concentric patterns, and symmetrical patterns (Erikson Institute Early Math Collaborative, 2014).

Repeating patterns contain a segment that continuously repeats. The segment that repeats is called the unit of repeat. It can vary in length and complexity but it is always the shortest string of elements that repeats. The unit of repeat can be thought of as the rule that governs a pattern.

Growing patterns increase or decrease by a constant amount. In growing patterns, instead of a segment or string of elements repeating, there is a repeating quantitative change. The counting system is an example of a growing pattern.

Unlike repeating and growing patterns, concentric patterns are non-linear and are often found in nature and other real world applications (e.g., clothing). A concentric pattern is made up of circles or rings that grow from a common center. The rings of a cross section of a tree trunk or the ripples that form when a rock is thrown into a pond are examples of concentric patterns.

Finally, symmetrical patterns have segments that repeat but extend outward in different directions from a line or point as if a mirror image.

RULES OF PATTERNS

Thinking more precisely involves figuring out the rule. This helps children to apply their thinking in different situations. We can begin with having children copy patterns, but the ability to copy patterns does not mean a child can identify the rule. We can help children focus on a pattern's rule by breaking the rule and encouraging children to complete it.

Activity – Identify the Rule

Participants create a new pattern and have their tablemates identify the rule of the pattern by either completing it (filling in a missing piece) or extending it. Participants are encouraged to consider creating one of the four different types of patterns. Participants share some of the rules of the patterns they created.

STRUCTURE AND FORMS OF PATTERNS

Identifying the structure and forms of patterns is a more abstract skill that develops over time for children enabling them to see connections and think about relationships outside of their physical representation.

Activity – Identifying Structure and Form

First, participants are asked to describe their patterns to their peers. Then they are asked to translate their pattern into another form (visual, auditory, movement, or spoken word) Finally, participants are asked which patterns that have been shared are the same and how they know they are the same.

BIG IDEAS OF PATTERN

After the participants have discussed and experienced patterns, they review how these activities relate to the three big ideas of pattern:

- Patterns are sequences (repeating or growing) governed by a rule; they exist both in the world and in mathematics;
- Identifying the rule of a pattern brings predictability and allows us to make generalizations; and
- The same pattern can be found in many different forms (Erikson Institute Early Math Collaborative, 2014, p. 83).

Activity – Video Analysis: Focus on the Child

Participants watch and reflect on video clips showing children presented with tasks involving pattern. While watching, participants consider the following questions to make connections to the big ideas.

- What big ideas about pattern do these children seem to understand?
- What big ideas about pattern do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring pattern might a teacher provide for these children to encourage them to develop their understanding further?

DEVELOPMENT OF CHILDREN'S THINKING ABOUT PATTERN

Clements and Sarama's (2014) learning trajectory for pattern and structure serves as the basis for discussion of the development of children's thinking around pattern. The table below outlines the developmental progression of children's understanding identified in the related learning trajectory (Clements & Sarama, 2014, pp. 221-225).

Age	Developmental Progression
2 years	Detects and uses patterning implicitly, but may not recognize sequential linear patterns explicitly or accurately.

3 years	Recognizes a simple pattern.
4 years	Fills in missing element of pattern. Duplicates AB pattern with a model. Continues AB repeating patterns. Duplicates simple patterns (not just along side the model pattern).
5 years	Extends simple repeating patterns.
6 years	Identifies the smallest unit of a pattern. Can translate patterns into new media.

EXPLORING PATTERN IN THE CLASSROOM

There are several things for teachers to keep in mind with regard to patterns and children's development of understanding of the concept. While visual patterns are the easiest, and rhythmic and movement patterns more difficult, it is still important to expose children to patterns in a variety of modes and orientations. Teachers should avoid promoting the misconception that patterns are only linear sequences of colors and shapes. Therefore, it is important that children are given opportunities to explore different forms of patterns including auditory, temporal, visual, and movement. When modeling patterns, adults should present at least three iterations (repetitions) of the unit of repeat and continue to verbalize the pattern past the last iteration to reinforce the idea that the pattern could go on indefinitely.

Teachers must consider materials and pattern complexity when working with preschool aged children. Early patterning experiences should include concrete materials that children can manipulate. Color is the most salient visual attribute followed by shape and size. When working with shape and size patterns, it is important to remove color as an attribute. As children progress in their understating of pattern, complexity can be increased by using materials with multiple attributes and by adjusting length of the unit of repeat and the number of elements included.

Children are interested in and spontaneously create patterns so adults should look for child-made patterns. Children's creation of patterns, however, does not necessarily mean that they understand the pattern's rule. Teachers can assess children's understanding by asking them to explain how their pattern "works." If a child responds to this question with a chanting response (e.g., "It goes blue, yellow, blue, yellow, blue, yellow..." until they run out of items), it indicates that he/she does not have good understanding generating rules to define the pattern. On the other hand, if a child responds with a unit-of-repeat response (e.g., "It is one blue and one yellow), it indicates he/she understands the rule. Questions can be used to extend children's thinking. What do you notice about this pattern? What part repeats? How could we name this pattern? What is the rule? How does this pattern continue? What comes next? Can you think of another pattern like this one? Describing and discussing patterns builds children's understanding.

Once again, the classroom environment provides many opportunities to explore pattern. It is the role of the teacher to highlight the math as it emerges in children's play and to offer feedback that helps them to see the underlying mathematics.

Activity – Carousel Brainstorming

This activity is designed to get participants thinking about the numerous ways patterns can be integrated into daily routines in the classroom. The chart papers with headings are distributed to each group along with the marker that correlates with the color the heading is written in. Each group chooses a recorder. The group has one minute to brainstorm a list of ways pattern can be incorporated into the topic. At the end of one minute, the groups rotate charts, taking their original color of marker with them. They read their new list and have another minute to add new ideas to the list. The groups continue to rotate until they return back to their original list. Each group highlights some of the ideas on the list and shares them with the whole group.

Activity – Video Analysis: Focus on the Lesson

Participants watch and reflect on a video clip of the "Who is Napping" lesson in which the teacher uses *The Napping House* (Wood, 1984) to explore the concept of patterns and have children describe the pattern in the story. While viewing, participants consider the following questions to examine three elements of the lesson: the children, the teacher, and the activity itself.

About the Children

- What big ideas about patterns do these children seem to understand?
- What big ideas about patterns do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?

About the Teacher

- What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    - About materials to use?
    - About questions to ask?
    - About space arrangement?
    - How does the teacher scaffold the children's thinking and explaining?

About the Activity

- How does this activity help children think about patterns?

- What modifications might you make if you were doing this activity in your classroom?
- What opportunities for exploring patterns might a teacher provide for these children to encourage them to develop their understanding further?

MAKING CONNECTIONS TO THE BIG IDEAS OF PATTERN

Connections are drawn between the big ideas of patterns and the learning standards, curriculum objectives and goals, and program standards that guide the work of preschool teachers in Iowa. The following table outlines the big ideas of mathematics for patterns and how they align to the various standards.

Big Ideas about Patterns (Erikson Institute Early Math Collaborative, 2014)	• Patterns are sequences (repeating or growing) governed by a rule; they exist both in the world and in mathematics; • Identifying the rule of a pattern brings predictability and allows us to make generalizations; and • The same pattern can be found in many different forms (p. 83).
Iowa Early Learning Standards (Early Childhood Iowa, 2012)	Standard: • Children understand patterns. Benchmarks: • Children recognize and create patterns moving from simple to complex. • Children predict what comes next in a pattern (p. 123).
Objectives for Development & Learning (Heroman et al., 2010)	Objective: • Demonstrates knowledge of patterns (p. 120)
NAEYC Early Childhood Program Standards (NAEYC, 2015) & Iowa Quality Preschool Program Standards (IQPPS) (Iowa Department of Education, 2006)	Standard: • Curriculum Criterion: • Children are provided varied opportunities and materials that help them to recognize and name repeating patterns (NAEYC 2.F.08, p. 18)

WHERE'S THE MATH IN BLOCKS (PART 2)

Activity – Block Riddle

Participants consider the following riddle: A preschooler says, "Teacher! I figured out that a triangle equals a square!" How can this be? Participants are given the opportunity to share their thoughts on the riddle. The presenter reveals that the solution revolves around the relationships in unit blocks.

Activity – Uncovering Relationships with Blocks

Participants work within their table groups to uncover the relationships between the unit blocks and the reason they are called unit blocks. Each group creates a poster explaining the relationships they uncover.

Activity – Gallery Walk

When all of the groups have finished creating their posters, the participants go on a gallery walk of the all of the posters. Participants are encouraged to use post-it notes to leave comments or questions for their colleagues. When the gallery walk is complete, the group discusses the experience and any relationships they had not considered before.

Activity – Video Analysis: Making a Block Sandwich

Participants watch and reflect on a video clip in which children are playing with unit blocks. While watching, participants consider the following questions to make connections to the big ideas and unit blocks.

- What, specifically, do these children say or do that gives you evidence of their thinking?
- How does the teacher scaffold the children's mathematical thinking?

Activity – Looking at Student's Building

Participants examine several photographs of block structures constructed by children and generate questions they would ask to help children think about the relationships among the blocks. Participants also discuss what comments they would make that would highlight the relationships among the blocks.

Activity – Pause & Reflect

Before leaving the session, participants are asked to pause and reflect on their learning and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing and stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom.

Session 6: What Kind of "Big" Is It? & Asking Questions and Finding Answers

Learning Objectives

As a result of attending this session, participants will be able to:

1. Summarize the big ideas of measurement, how children's understanding of measurement develops, and ideas for exploring measurement in the classroom.
2. Summarize the big ideas of data analysis, how children's understanding of data analysis develops, and ideas for exploring data analysis in the classroom.

Materials

- Pairs of paper strips cut into various lengths (should be same width and color)
- Collections of miscellaneous materials for ordering

Participant Handouts (see Appendix G)

- Note-Catcher handout
- Note-Catcher Video Analysis handouts for Measurement and Data Analysis
- "Rulers" from *Wally's Stories*: *Conversations in the Kindergarten* (Paley, 1981, pp. 13-19) handout
- Learning Trajectory for Measurement and Data Analysis handout
- Reflection handout

MEASUREMENT: WHAT KIND OF "BIG"IS IT?

Activity – Who's Just Right for Me?

Participants find the person whose strip is exactly the same length as theirs. The duo sits down together and talks about how you used measurement this morning. After discussing for several minutes pairs share some of the ways they used measurement.

Activity – Ordering Objects by Size

In table groups, participants arrange a set of objects for smallest to biggest. Once the items are arranged, the group makes a list of the items numbered from 1 for the smallest up to whatever number needed for the biggest. Then the group re-arranges the objects from smallest to biggest in a different way. Once again, they number the order of the arrangement. The presenter then leads a discussion regarding how the groups ordered their sets stressing the use of attributes and comparison.

Activity – Let's Draw a Ruler

The presenter asks the group what is needed to represent a ruler on the board and draws a ruler according to the directions of the group. While drawing, the presenter makes comments and asks questions regarding the directions of the group to emphasize the importance of precision in measurement. This exercise leads to a discussion of the rules of rulers and what rulers are used for.

Activity –Reader's Theater Wally's Stories: Rulers

Nine participants are selected as readers for one of the roles in the reader's theater. After the reading, the group discusses what the scenario in this reading indicates regarding children's understanding of measurement.

BIG IDEAS OF MEASUREMENT

After the participants have discussed and experienced measurement, they review how these activities relate to the three big ideas of measurement:

- Many different attributes can be measured, even when measuring a single object;
- All measurement involves "fair" comparison; and
- Quantifying a measurement helps us to describe and compare more precisely (Erikson Institute Early Math Collaborative, 2014, p. 99).

Activity – Video Analysis: Focus on the Child

Participants watch and reflect on video clips showing children presented with tasks involving measurement. While watching, participants consider the following questions to make connections to the big ideas.

- What big ideas about measurement do these children seem to understand?
- What big ideas about measurement do these children seem to be learning?

- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring measurement might a teacher provide for these children to encourage them to develop their understanding further?

DEVELOPMENT OF CHILDREN'S THINKING ABOUT MEASUREMENT

Clements and Sarama's (2014) learning trajectory for length measurement as well as the learning trajectory for volume measurement serve as the basis for discussion of the development of children's thinking around measurement. The table below outlines the developmental progression of children's understanding identified in the two related learning trajectories (Clements & Sarama, 2014, pp. 194-197, pp. 207-.

Age	Developmental Progression
2 years	Intuitively compares, orders, and builds with materials. May use gesture to indicate attribute because they lack vocabulary for specific dimensions.
3 years	Identifies length as attribute. Often understand length as an absolute (e.g., I am tall), but not as a comparison (e.g., I am taller than my brother but shorter than my sister). May compare not-corresponding parts of shape in determining side length. Identifies capacity or volume as an attribute.
4 years	Physically aligns two objects to determine which is longer, taller, holds more, and so on. May be able to measure with a rule, but often lacks understanding or skill (e.g., ignores starting point). Fills a container using another (smaller container) and counts the number needed to completely fill the container.
5 years	Begins to use indirect comparison. Shows an interest in assigning numbers to measures.
6 years	Orders lengths, marked in 1 to 6 units. Lays units end to end. May not recognize the need for equal length units. Able to estimate number of scoops need to fill a container. Able to attend to both the portion of container filled with the portion remaining unfilled. Recognizes when a container is half full.

EXPLORING MEARSUREMENT IN THE CLASSROOM

Measurement is a complex combination of concepts that develop over time so the teaching of measurement should not be oversimplified. Teachers must give children opportunities to explore the ideas of measurement. One way in which this can be done is through the use of informal activities that focus children's attention on size attributes and direct comparison of objects. Children should also be encouraged to solve real measurement problems. Real world problems regarding measurements will naturally lead them to think about quantifying measures and the need for units to count. When using non-standard units of measurement, use manipulative units that closely connect with measurement tools. Attend to precision in measurement by incorporating rich, descriptive language. Descriptive words such as longer, heavier, wider, taller are more precise than bigger so adults in early childhood classrooms should carefully consider the comparative adjectives used when talking about measurement with children.

Activity – Video Analysis: Focus on the Lesson

Participants watch and reflect on a video clip of the "Just Right for Me" lesson in which children compare the size of their hands to items found in the classroom. While viewing, participants consider the following questions to examine three elements of the lesson: the children, the teacher, and the activity itself.

About the Children

- What big ideas about measurement do these children seem to understand?
- What big ideas about measurement do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?

About the Teacher

- What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    - About materials to use?

- About questions to ask?
- About space arrangement?

- How does the teacher scaffold the children's thinking and explaining?

About the Activity

- How does this activity help children think about measurement?
- What modifications might you make if you were doing this activity in your classroom?
- What opportunities for exploring measurement might a teacher provide for these children to encourage them to develop their understanding further?

MAKING CONNECTIONS TO THE BIG IDEAS OF MEASUREMENT

Connections are drawn between the big ideas of measurement and the learning standards, curriculum objectives and goals, and program standards that guide the work of preschool teachers in Iowa. The following table outlines the big ideas of mathematics for measurement and how they align to the various standards.

Big Ideas about Measurement (Erikson Institute Early Math Collaborative, 2014)	• Many different attributes can be measured, even when measuring a single object; • All measurement involves "fair" comparison; and • Quantifying a measurement helps us to describe and compare more precisely (p. 99).
Iowa Early Learning Standards (Early Childhood Iowa, 2012)	Standard: • Children understand comparisons and measurement. Benchmarks: • Children measure objects using non-standard units of measurement. (p. 132).
Objectives for Development & Learning (Heroman et al., 2010)	Objective: • Compares and measures.
NAEYC Early Childhood Program Standards (NAEYC, 2015) &	Standard: • Curriculum Criterion:

Iowa Quality Preschool Program Standards (IQPPS) (Iowa Department of Education, 2006)	• Children are provided varied opportunities and materials to help them understand the concept of measurement by using standard and non-standard units of measurement (NAEYC 2.F.05, p. 17/IQPPS 2.24, p. 29).

DATA ANALYSIS: ASKING QUESITONS AND FINDING ANSWERS

Activity – How Do You Data

Participants discuss with a partner how they use data in their daily lives.

DATA ANALYSIS

The Erikson Institute Early Math Collaborative (2014) stated data analysis involves gathering information in a quantitative way, and organizing it in a way that makes comparison and generalization possible. Through the process of analyzing the data, we learn something new.

For authentic problem solving to take place, there must be an authentic problem – one whose solution is not obvious or predetermined. If the data and results are obvious and predictable, collecting and analyzing is mechanical and no real problem solving or thinking is involved. For the purpose of exploring the big ideas of data analysis, the fictional problem for this session centers on what kinds of pizza to order for a class party.

Activity – Data Collection and Analysis

For the pizza problem, consider how we could collect data on what kinds of pizza everyone prefers. Possible methods include direct comparison, photos, name cards, or tally marks. Once the group generates a data collection method, the data is assembled and then the group considers how to connect the data to quantities. One of the better ways to organize raw data to examine quantities with children is to use a bar graph. The final step involves analysis of the data to draw conclusions. This is a critical step because it is where new information is learned and it makes the process make sense for young children. As a general guideline, the analysis of the data should have twice as much time devoted to it as was devoted to the collection of the data. It is through experience that children begin to understand that collecting data is not a popularity contest. Considering the data for the pizza problem, participants make comparisons and draw conclusions based on the data.

BIG IDEAS OF DATA ANALYSIS

After the participants have discussed and experienced data analysis, they review how these activities relate to the three big ideas of data analysis:

- The purpose of collecting data is to answer questions when the answers are not immediately obvious;
- Data must be represented in order to be interpreted, and how data are gathered and organized depends on the question; and
- It is useful to compare parts of the data and to draw conclusions about the data as a whole (Erikson Institute Early Math Collaborative, 2014, p. 113).

Activity – Video Analysis: Focus on the Child

Participants watch and reflect on video clips showing children presented with tasks involving data analysis. While watching, participants consider the following questions to make connections to the big ideas.

- What big ideas about data analysis do these children seem to understand?
- What big ideas about data analysis do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring data analysis might a teacher provide for these children to encourage them to develop their understanding further?

DEVELOPMENT OF CHILDREN'S THINKING ABOUT DATA ANALYSIS

Clements and Sarama (2014) stressed that the foundations of data analysis are developed throughout preschool and kindergarten years through other areas, such as counting and classification. While not as detailed as other learning trajectories, the table below outlines young children's development of understanding of data analysis (Clements & Sarama, 2014, p. 225-226).

Developmental Progression
Initially does not use categories to collect data.
Classifies responses and represents data by categories.
Uses physical objects, then picture graphs, then line plots, and finally bar graphs.
Organizes and displays data through both numerical summaries (tallies, counts, etc.) and graphical displays. Compares parts of the data, makes statements about the data as a whole, and determines whether the graphs answer the initial questions (by second grade).

EXPLORING DATA ANALYSIS IN THE CLASSROOM

As mentioned in the discussion of the big ideas of data collection, children should be engaged in collecting data for a reason rather than collecting the data just because. It is important that teachers design data investigations that deepen understanding for children. An extremely important thing for teachers to consider is how valuable it is to involve children in determining how data is collected and represented. The type of investigation being conducted leads to the data collection method. Two types of methods that fit well in early childhood classroom are inventories and surveys (Erikson Institute Early Math Collaborative, 2014). An inventory would begin with the sorting and counting of categories in a collection and then representing results in graph or chart form. The shoe sort activity that was done in the first session is an example of this method. Other investigations lend themselves more to the use of a survey to collect data. Surveys could be used in a fact-finding investigation (e.g., How many children are at school today? or How do you get to school?) or to collect information on preference (e.g., What is your favorite kind of pizza?) Surveys can also be used connect to classroom investigations and support a home-school connection.

Another important element to consider when exploring data analysis with children is the form of representation that is used to organize the data. The most concrete form of data representation is an object graph but it is important to organize objects using some type of grid to control for variations in size that would make comparisons challenging. An inexpensive grid can be made with a plastic shower curtain that is marked off in a grid. Pictographs can extend from object graphs by having children draw a picture of the object on a post-it note and securing it to the grid. This allows for the data to be preserved when the objects cannot remain in graph indefinitely (e.g., shoes on the shoe graph).

When exploring data analysis with young children, teachers should support the language of data analysis by encouraging children to label and describe parts of data displays so that others understand the data display. Language can also incorporated into the data collection by asking the same question or responding to a sentence stem.

Activity – Video Analysis: Focus on the Lesson

Participants watch and reflect on a video clip of the "Shoe Graph" lesson in which children create a shoe graph to organize data from their shoe sort. While viewing, participants consider the following questions to examine three elements of the lesson: the children, the teacher, and the activity itself.

<u>About the Children</u>

- What big ideas about data analysis do these children seem to understand?
- What big ideas about data analysis do these children seem to be learning?

- What, specifically, do these children say or do that gives you evidence of their thinking?

About the Teacher

- What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
  - About materials to use?
  - About questions to ask?
  - About space arrangement?
- How does the teacher scaffold the children's thinking and explaining?

About the Activity

- How does this activity help children think about data analysis?
- What modifications might you make if you were doing this activity in your classroom?
- What opportunities for exploring data analysis might a teacher provide for these children to encourage them to develop their understanding further?

MAKING CONNECTIONS TO THE BIG IDEAS OF DATA ANALYSIS

Connections are drawn between the big ideas of data analysis and the learning standards, curriculum objectives and goals, and program standards that guide the work of preschool teachers in Iowa. The following table outlines the big ideas of mathematics for data analysis and how they align to the various standards.

Big Ideas about Data Analysis (Erikson Institute Early Math Collaborative, 2014)	• The purpose of collecting data is to answer questions when the answers are not immediately obvious; • Data must be represented in order to be interpreted, and how data are gathered and organized depends on the question; and

	• It is useful to compare parts of the data and to draw conclusions about the data as a whole (p. 113).
Iowa Early Learning Standards (Early Childhood Iowa, 2012)	Standard: • Children understand comparisons and measurement. Benchmark: • Children make comparisons between several objects based on one or more attributes (p. 132).
Objectives for Development & Learning (Heroman et al., 2010)	Objective: • Uses inquiry skills • Represents his or her thinking through drawing, dramatizing, graphing, or making models (p. 126)
NAEYC Early Childhood Program Standards (NAEYC, 2015) & Iowa Quality Preschool Program Standards (IQPPS) (Iowa Department of Education, 2006)	Standard: • Curriculum Criterion: • Children are provided varied opportunities and materials to collect data an to represent and document their findings (e. g., through drawing or graphing (NAEYC 2.G.05, p. 19)

Activity – Pause & Reflect

Before leaving the session, participants are asked to pause and reflect on their learning and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing and stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom.

Session 7: Mapping the World around Us & Where's the Math in Blocks (Part 3)

Learning Objectives

As a result of attending this session, participants will be able to:

1. Summarize the big ideas of spatial relationships, how children's understanding of spatial relationships develops, and ideas for exploring spatial relationships in the classroom.
2. Describe how blocks support children's mathematical thinking and how teachers can scaffold children's thinking during block building experiences.

Materials

- Unit blocks

Participant Handouts (see Appendix H)

- Note-Catcher handout
- Note-Catcher Video Analysis handout
- Learning Trajectory for Spatial Thinking handout
- "A Developmental Look at a Rigorous Block Play Program" article (Tepylo, Moss, & Stephenson, 2015)
- Reflection handout

SPATIAL RELATIONSHIPS: MAPPING THE WORLD AROUND US

Activity – How do you use spatial relationships?

Participants discuss with a partner how they use spatial relationships in a typical day or week and which of those applications are particularly mathematical.

Spatial relationships help us know where we are and how to get from one place to another. They also help us to visualize representations of real-world objects. Preschoolers can represent space by describing relationships between objects and locations with words and gestures, as well as by drawing maps and constructing models. These concepts are mathematical because of their relational nature. Experiences talking about, organizing, moving through, drawing and modeling space provide a conceptual foundation for children's later mathematical learning (Erikson Institute Early Math Collaborative, 2014).

Activity – Let's Draw a Map

The presenter asks the group what is needed to represent this classroom on the board and draws the map according to the directions of the group. This exercise leads to a discussion regarding precision. Like many of the concepts addressed over the course of these sessions, the key to getting the math out of representations of places and objects is discussion. Children need opportunities to develop vocabulary for describing spatial relationships. Therefore, it is important that adults model precise spatial language. Instead of responding to a question about where something is located within the classroom with a hand gesture and "It's over there," adults can use precise language. Say, "The paint shirts are on the hooks, next to the cupboard, under the window," to model precise spatial language for young children. Encouraging children to draw pictures and maps and move through space also is important to developing mathematical precision with regard to spatial relationships.

Activity – Experiencing Perspective

Participants are given a collection of two-and three-dimensional shapes and asked to group them according to their similarities. Once they have grouped their items, the groups share how they sorted the items. This conversation leads to a discussion on perspective. Young children are beginning to develop an awareness of perspective, the understanding that spatial relationships look different when viewed from different positions.

BIG IDEAS OF SPATIAL RELATIONSHIPS

After the participants have discussed and experienced spatial relationships, they review how these activities relate to the three big ideas of spatial relationships:

- Relationships between objects and places can be described with mathematical precision;
- Our own experiences of space and two-dimensional representations of space reflect a specific point of view; and
- Spatial relationships can be visualized and manipulated mentally (Erikson Institute Early Math Collaborative, 2014, p. 131).

Activity – Video Analysis: Focus on the Child

Participants watch and reflect on video clips showing children presented with tasks involving spatial relationships. While watching, participants consider the following questions to make connections to the big ideas.

- What big ideas about spatial relationships do these children seem to understand?
- What big ideas about spatial relationships do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring spatial relationships might a teacher provide for these children to encourage them to develop their understanding further?

DEVELOPMENT OF CHILDREN'S THINKING ABOUT SPATIAL RELATIONSHIPS

Clements and Sarama's (2014) learning trajectory for spatial thinking, which includes spatial orientation as well as spatial visualization and imagery, serves as the basis for discussion of the development of children's thinking around spatial relationships. The table below outlines the developmental progression of children's understanding identified in the learning trajectory (Clements & Sarama, 2014, pp. 137-140).

Age	Developmental Progression
0-2 years	Understands initial vocabulary of spatial relations and location. Can move shapes to a location.
2-3 years	Uses landmarks to find objects or location near them if object is specified ahead of time. Can move shapes to a location.
4 years	Locates objects after movement if target is not specified and searches areas comprehensively often in a circular search pattern. Mentally turns object in easy tasks.
5 years	Locates objects after moving relating locations separate from own position. Represents objects' positions relative to landmarks. Keeps track of own location in open areas or mazes. Uses simple coordinate labels in some situations. Knows a shape has to be flipped to match another shape, but flips it the wrong direction.
6 years	Locates objects using maps with pictorial cues. Knows a shape must be turned a certain degree to fit into a puzzle.

EXPLORING SPATIAL RELATIONSHIPS IN THE CLASSROOM

Exploring spatial relationships in early childhood classrooms begins with supporting the language of spatial relationships. Teachers should use spatial vocabulary to bring attention to spatial relationships (e.g., in, on, under, up and down; beside and between; in front of and behind; left and right). The use of this vocabulary encourages greater precision in describing spatial relationships. As it had been discussed throughout this course, teachers should also acknowledge receptive understanding (e.g., gestures, movements, and other forms of representation) of spatial relationships.

The power of blocks in preschool classrooms has been explored throughout this series. The goal of these sessions (including the third and final part later in this session, is to encourage teachers to harness the power of block building. Building with blocks provides a powerful, playful way for children to explore spatial relationships. To support this, every classroom should be well stocked with blocks and provide ample space and time for building. The second session provided an overview of the stages of block building so that teachers can recognize children's stages and encourage them to move to the next.

Differences in experiences and expectations lead to gender differences in spatial ability that begin to emerge before kindergarten (Erikson Institute Early Math Collaborative, 2014). To ensure that this does not continue, teachers should ensure equal access to activities that promote the development of understand in spatial relationships for all children. Because girls may not gravitate to types of activities that promote visual-spatial skills, teachers should create spaces that encourage them to engage with construction or building materials

A variety of common materials, activities, and routines often found in early childhood classrooms build understanding of spatial relationships including blocks and construction materials like unit blocks, plastic bricks, and magnetic tiles. Movement songs and games such as Hokey Pokey, Simon Says, Captain May I? and informal games and activities like Where Is It?; Doggie, Doggie, Where's My Bone? also teach spatial vocabulary. Also included are obstacle courses and mapping activities; and puzzles such as interlocking puzzles, pattern blocks, and tangrams.

Activity – Video Analysis: Focus on the Lesson

Participants watch and reflect on a video clip of the "Walk with Rosie" lesson in which children navigate through an obstacle course and then map it as a follow up to reading *Rosie's Walk* (Hutchins, 1968/1986). While viewing, participants consider the following questions to examine three elements of the lesson: the children, the teacher, and the activity itself.

About the Children

- What big ideas about spatial relationships do these children seem to understand?
- What big ideas about spatial relationships do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?

About the Teacher

- What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    - About materials to use?
    - About questions to ask?
    - About space arrangement?
- How does the teacher scaffold the children's thinking and explaining?

About the Activity

- How does this activity help children think about spatial relationships?
- What modifications might you make if you were doing this activity in your classroom?
- What opportunities for exploring spatial relationships might a teacher provide for these children to encourage them to develop their understanding further?

MAKING CONNECTIONS TO THE BIG IDEAS OF SPATIAL RELATIONSHIPS

Connections are drawn between the big ideas of spatial relationships and the learning standards, curriculum objectives and goals, and program standards that guide the work of preschool teachers in Iowa. The following table outlines the big ideas of mathematics for spatial relationships and how they align to the various standards.

Big Ideas about Spatial Relationships (Erikson Institute Early Math Collaborative, 2014)	• Relationships between objects and places can be described with mathematical precision; • Our own experiences of space and two-dimensional representations of space reflect a specific point of view; and • Spatial relationships can be visualized and manipulated mentally (p. 131).
Iowa Early Learning Standards (Early Childhood Iowa, 2012)	Standard: • Children understand shapes and spatial relationships. Benchmarks: • Children demonstrate understanding of spatial words such as up, down, over, under, top, bottom, inside, outside, in front, and behind (p. 125)
Objectives for Development & Learning (Heroman et al., 2010)	Objective: • Explores and describes spatial relationships and shapes. • Understands spatial relationships (p. 112)
NAEYC Early Childhood Program Standards (NAEYC, 2015) & Iowa Quality Preschool Program Standards (IQPPS) (Iowa Department of Education, 2006)	Standard: • Curriculum Criterion: • Children are provided varied opportunities and materials that encourage them to integrate mathematical terms into everyday conversation (NAEYC 2.F.04, p. 17).

WHERE'S THE MATH IN BLOCKS (PART 3)

Activity – Building a Bridge

Participants work in teams to build a bridge that crosses over the river that runs through the room. As they are building, the teams keep track of the mathematical thinking they engage in. Once the groups have completed their bridges, each group shares their bridge structure and the mathematical thinking they used as they constructed it.

Activity – Video Analysis and Photograph Analysis

Participants view a portion of Ken Burn's movie, *Brooklyn Bridge*, in which children in a first grade classroom are constructing the Brooklyn Bridge. Participants also view photographs of different stages of building. After seeing both the video and photos, participants discuss children's mathematical thinking use in the construction of the bridge.

Activity – Video Analysis: Taller Tower

Participants watch and reflect on a video clip in which children are playing with unit blocks. While watching, participants consider the following questions to make connections to the big ideas and the use of blocks.

- What, specifically, do these children say or do that gives you evidence of their thinking?
- How does the teacher scaffold the children's mathematical thinking?
- How do blocks promote indirect comparison?

Activity – Text Rendering Protocol – "Rigorous Block Play Program" article

The purpose of the experience is to collaboratively construct meaning, clarify, and expand thinking about the article. Each participant reads the article, "A Developmental Look at a Rigorous Block Play Program" (Tepylo, Moss, & Stephenson, 2015), and notes the sentence, the phrase, and the word that he/she thinks is particularly important. Three rounds of sharing are done in table groups. In the first round of sharing, each person shares a sentence that he/she believes is significant. In the second round, each person shares a phrase that he/she believes is significant. In the third round, each person shares the word that he/she believes is significant. The table groups discuss the ideas and themes that emerged during sharing and how the article confirms and/or challenges their own practices. As a large group, the presenter records the words that emerged and the whole group discusses new insights taken from the article.

Activity – Pause & Reflect

Before leaving the session, participants are asked to pause and reflect on their learning and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing and stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom.

Session 8: The Shape of Things & Good Math in Good Books

Learning Objectives

As a result of attending this session, participants will be able to:

1. Summarize the big ideas of shape, how children's understanding of shape develops, and ideas for exploring shape in the classroom.
2. Describe characteristics to consider in counting books and how books support the big ideas the big ideas of mathematics and the development of children's mathematical thinking.

Materials

- Paper shapes
- Three-dimensional shapes

Participant Handouts (see Appendix I)

- Note-Catcher handout
- Note-Catcher Video Analysis handout
- Learning Trajectory for Shapes handout
- Good Math in Good Books handouts
- Reflection handout

SPATIAL RELATIONSHIPS: MAPPING THE WORLD AROUND US

Activity – Match and Name Shapes

Participants find the person whose shape is exactly the same as theirs other than the color. Pairs discuss how they know they are exact matches, name the shape and share it with two other pairs near them.

Activity – Discussion – What Shape Is It?

The presenter shares an example from Clements and Sarama (2014) of a scenario that took place between a teacher and a little boy in a kindergarten classroom in which the little boy names a triangle correctly and explains that he knows it is a triangle because it has "three straight sides and three angles." When presented with a triangle in a different orientation the same boy says it is not a triangle. The teacher talks with the little boy and helps him to recall what he had said earlier were the characteristics of a triangle. The boy agreed that the inverted triangle did, in fact, have three straight sides and three angles but he insisted it was not a triangle because it was upside down. Participants discuss whether or not the child in this example knows triangle or not and what is driving his thinking about triangles.

According to Clements and Sarama (2014), shape is a fundamental concept in cognitive development. It is fundamental in geometry, which is one of the weakest topics in mathematics for students in the United States. Preschool children in the United States know less about shape than their counterparts in other countries. Even many adults have only a very basic knowledge of shape.

Teachers in early childhood classrooms do not need to be extremely technical when it comes to the concept of shape. What is important is that teachers deepen their knowledge of two- and three-dimensional shapes, how they are defined, how they relate to one another, and subtle distinctions and rules regarding shape. This knowledge helps teachers to highlight children's discoveries and guide their experiences.

DEFINING SHAPES

Even when working with young children, it is necessary to move beyond superficial shape labels and encourage children to recognize and define the attributes, i.e., the characteristics, of shapes. This involves precision, which is often not immediately obvious to young children. Defining attributes include the number of sides, the length of sides, the size of angles, flat or curved sides, and points (corners). Color, size, and positioning are examples on non-defining attributes.

Activity – Shape Primer

Participants read through the "Shape Primer" (Erikson Institute Early Math Collaborative, 2014, pp. 163-166), which defines shape categories for two- and three-dimensional shapes. The shape primer also provides examples of most common and less common types of shapes in each category. After reviewing the primer, participants identify the most useful piece of information they found and share with their tablemates.

BIG IDEAS OF SHAPE

After the participants have discussed and experienced shape, they review how these activities relate to the three big ideas of shape:

- Shapes can be defined and classified by their attributes;
- The flat surfaces of solid (three-dimensional) shapes are two-dimensional shapes; and

- Shapes can be combined and separated (composed and decomposed) to make new shapes (Erikson Institute Early Math Collaborative, 2014, p. 147).

Activity – Video Analysis: Focus on the Child

Participants watch and reflect on video clips showing children presented with tasks involving shape. While watching, participants consider the following questions to make connections to the big ideas.

- What big ideas about shape do these children seem to understand?
- What big ideas about shape do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring shape might a teacher provide for these children to encourage them to develop their understanding further?

DEVELOPMENT OF CHILDREN'S THINKING ABOUT SHAPE

Clements and Sarama's (2014) learning trajectory for shapes serves as the basis for discussion of the development of children's thinking with regard to shape. The table below outlines the developmental progression of children's understanding identified in the learning trajectory (Clements & Sarama, 2014, pp. 157-164).

Age	Developmental Progression
0-2 years	Compares real-world objects. Matches familiar shapes (circle, square, typical triangle) with *same size and orientation.* Matches familiar shapes with *different sizes.* Matches familiar shapes with *different orientations.*
3 years	Recognizes and name typical circle, square, and less often, triangle. Judges two shapes the same if they are more visually similar than different
3-4 years	Matches wider variety of shapes with *different sizes and orientations*. Matches combinations of shapes to each other.
4 years	Recognizes less typical squares and triangles and maybe some rectangles, but not rhombuses. Says two shapes are the same after matching one side on each. Uses manipulatives to represent parts of shapes. Looks for different

	attributes, but may only examine parts of shape.
4-5 years	Recognizes more rectangle sizes, shapes, and orientations. Identifies sides as distinct geometric objects. Looks for differences in attributes. Recognizes angles as separate geometric objects, in the context of corners.
5 years	Recognize most familiar shapes and typical examples of other shapes (hexagon, rhombus, and trapezoid).
6 years	Names most common shapes. Recognizes right angles to distinguish rectangle from parallelogram.

EXPLORING SHAPE IN THE CLASSROOM

When exploring in early childhood classrooms teachers should provide diverse examples of shape rather than focusing only on the more typical, preferred representations of shape. It is also important to explore both two- and three-dimensional shapes with preschool aged children. Teachers should examine their classroom materials for examples of two- and three-dimensional shapes.

One of the common themes throughout these sessions has been the need to provide children with the information needed to move them toward precision in describing mathematical thinking. This is true for the mathematics of shape as well. Children tend to overgeneralize and connect shape to things in their world that are not accurate representations of shape. For example, a child might say a slice of pizza is a triangle or a flowerpot is a cylinder. It is important for teachers to acknowledge and embrace children's enthusiasm but give them information to get their definitions as "right" as possible. One way to do this would be to discuss both examples and non-examples of shapes.

Many activities and materials found in early childhood classrooms build children's understanding of shape including match and name shapes (opening activity), shape feely bag, "I Spy" shapes, shape hunts, pattern blocks, puzzles, tangram puzzles, and geoboards.

Activity – Video Analysis: Focus on the Lesson

Participants watch and reflect on a video clip of the "Feel for Shapes" lesson in which children are distinguishing shapes by touch rather than by sight. While viewing, participants consider the following questions to examine three elements of the lesson: the children, the teacher, and the activity itself.

About the Children

- What big ideas about shape do these children seem to understand?
- What big ideas about shape do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?

About the Teacher

- What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    - About materials to use?
    - About questions to ask?
    - About space arrangement?
- How does the teacher scaffold the children's thinking and explaining?

About the Activity

- How does this activity help children think about shape?
- What modifications might you make if you were doing this activity in your classroom?
- What opportunities for exploring shape might a teacher provide for these children to encourage them to develop their understanding further?

MAKING CONNECTIONS TO THE BIG IDEAS OF SHAPE

Connections are drawn between the big ideas of shape and the learning standards, curriculum objectives and goals, and program standards that guide the work of preschool teachers in Iowa. The following table outlines the big ideas of mathematics for shape and how they align to the various standards.

Big Ideas about Shape (Erikson Institute Early Math Collaborative, 2014)	• Shapes can be defined and classified by their attributes; • The flat surfaces of solid (three-dimensional) shapes are two-dimensional shapes; and • Shapes can be combined and separated (composed and decomposed) to make new shapes (p. 147).
Iowa Early Learning Standards (Early Childhood Iowa, 2012)	Standard: • Children understand shapes and spatial relationships. Benchmarks: • Children identify 2- and 3-dimensional shapes. • Children notice characteristics, similarities, and differences among shapes, such as corners, points, edges, and sides. • Children notice how shapes fit together and be taken apart to form other shapes (p. 125).
Objectives for Development & Learning (Heroman et al., 2010)	Objective: • Explores and describes spatial relationships and shapes. • Understands shapes (p. 112)
NAEYC Early Childhood Program Standards (NAEYC, 2015) & Iowa Quality Preschool Program Standards (IQPPS) (Iowa Department of Education, 2006)	Standard: • Curriculum Criterion: • Children are provided varied opportunities and materials to understand basic concepts of geometry by; for example, by naming and recognizing two- and three-dimensional shapes and recognizing how figures are composed of different shapes (NAEYC 2.F.06, p. 18/IQPPS 2.26, p. 28).

GOOD MATH IN GOOD BOOKS

Activity – Story Acting – *Mouse Count*

The presenter chooses a participant to play the role of the snake in the book *Mouse Count* (Walsh, 1991). As the presenter reads the book, the participant playing the snake chooses other participants as mice to put into a jar which is in an area designated by a piece of rope on the floor.

Activity – Video Analysis

Participants watch and reflect on a video clip of the "Mouse Collections" lesson. While watching, participants consider the role of the book in meaning making for the children.

Activity – Counting Book Analysis

Participants examine the counting books on the tables throughout the room and consider the following questions:

- How high does the book count to? (1-10, higher)
- As the numbers change, are the illustrations cumulative (more join or leave the same group) or is each number a new set of things?
- Are any hierarchical relationships depicted in the illustrations (one smaller number embedded in a larger number)?
- Is zero used appropriately?
- If involving higher numbers, does the book introduce patterns or arrays or somehow reinforce the idea of grouping and place value?
- Does the book introduce separating and joining concepts (addition and subtraction)? Does it do so at fairly simple level of counting up or down by ones? Are counting-on strategies possible?
- Is the counting embedded in a story that helps make a math all around us connection?
- Is the counting tied to another informational concept, such as animal study?

After analyzing the counting books, each group selects the one most mathematically powerful counting book to share with the whole group.

Activity – Finding Good Math in Good Books

Partners select one non-counting book and find the math within using the following questions from the handout as a guide:

- What's the Math? Identify specific BIG IDEAS of Math that this book might be well suited to use to introduce or develop understanding for the children in your classroom.
- What are some open-ended discussion questions or problem situations I can ask that will trigger mathematical thinking or understanding?
- What are some extending activities that will allow children to develop and construct mathematical understanding?

Activity – Pause & Reflect

Before leaving the session, participants are asked to pause and reflect on their learning and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing and stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom.

Session 9: Big Connections – Math and Mindset

Learning Objectives

As a result of attending this session, participants will be able to:

1. Describe connections between the big ideas and the connections to classroom practices.
2. Define mindset and describe how it affects learning and growth for both teachers and children particularly in relation to mathematics.

Materials

- Cards with representing 1 through 25 in different ways (dots, tally marks, numerals, Roman numerals)
- Pattern blocks
- Each of the 26 big ideas of mathematics on a strip of paper

Participant Handouts (see Appendix J)

- Note-Catcher handout
- Quilt Patterns handout
- "Preschoolers Grow Their Brains: Shifting Mindsets for Greater Resiliency and Better Problem Solving" article (Pawlina & Stanford, 2011)
- Reflection handout

BIG CONNECTIONS TO THE BIG IDEAS

Activity – Who's Next

Participants receive an index card with a representation of a number on it. The cards are numbered from 1 to 25 using different representations including numerals, dots, tally marks, and Roman numerals. For larger groups, the cards would be numbered to the equivalent to the number of people in the session. Participants are asked to line up in order around the room.

Activity – Quilt Patterns

The presenter describes a scenario to the group in which someone wishes to make a quilt and they have two patterns to choose from. The person wants to make the larger of the two patterns and therefore needs to determine which one covers more area. Using the pattern blocks, the participants solve the problem individually. Once they have solved the problem individually, participants share strategies and justify their solutions within their table groups and then discuss the following question: What is the math in this task?

Activity – Big Ideas/Big Connections

For this activity, participants work in groups of three or four. Each group chooses two of the big ideas, discusses how those two big ideas are connected, and creates a poster that tells the story of the connections between the big ideas they selected. Each group describes a classroom episode that touches on both big ideas. When the groups have completed their posters, the group goes on a gallery walk to view each poster. The large group processes the activity discussing what they noticed in the connections and in the classroom episodes, how they are similar, and how they are different.

MATH AND MINDSET

Activity – Video: What is Mindset?

Participants view a video of Dr. Carol Dweck describing the growth mindset and discuss their thoughts afterward.

Activity – Text Rendering Protocol – "Preschoolers Grow Their Brains" article

The purpose of the experience is to collaboratively construct meaning, clarify, and expand thinking about the article. Each participant reads the article, "Preschoolers Grow Their Brains: Shifting mindsets for greater resiliency and better problem solving" (Pawlina & Stanford, 2011) and notes the sentence, the phrase, and the word that he/she thinks is particularly important. Three rounds of sharing are done in table groups. In the first round of sharing, each person shares a sentence that he/she believes is significant. In the second round, each person shares a phrase that he/she believes is significant. In the third round, each person shares the word that he/she believes is significant. The table groups discuss the ideas and themes that emerged during sharing and how the article confirms and/or challenges their own practices. As a large group, participants discuss what they hear from others in the group, what it means for their work, and how it help them understand what mindset is about, particularly in relation to math. Finally, participants discuss how mindset affects the learning and growth of themselves, teachers with whom they work, and the young children with whom they work.

Activity – Mindset Interview

Each participant finds a partner to interview and asks these questions:

- In any area of your life, do you have a growth mindset? Why did it develop? What has it allowed you to do?
- How would you describe your mindset toward math? Has your math mindset shifted over the course of these sessions?

Activity – Pause & Reflect

Before leaving the session, participants are asked to pause and reflect on their learning and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing and stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom.

Site-based Coaching

An important element of this project that supports the implementation of teachers' new learning into classroom practice is ongoing site-based coaching. Between professional development sessions, teachers will participate site-based coaching sessions with early childhood consultants that have been trained in the big ideas of mathematics. The focus of the coaching sessions is on planning, observation, and reflection on mathematics activities that teachers implement in their classrooms. This practice not only deepens teachers' understanding of the mathematics content, but also supports them in teaching the content to young children through learning experiences that appropriate and meaningful for them.

In the planning phase of the coaching session the teacher and coach work together to intentionally plan a mathematics activity to be implemented in the classroom that relates to the content topic area and the big ideas that were present in the prior professional development session. The following elements are included in the plan: the content topic area addressed (i.e., sets and sorting, number sense, counting, number operations, patterns, measurement, data analysis, spatial relationships, or shape); the big ideas that children will explore; the learning standards and objectives that will be addressed; the delivery format for the lesson (i.e., whole group, small group, or centers/free choice); the materials needed; the procedures for the lesson including specific strategies that will be used; the key vocabulary words to be highlighted; the questions that will be used to assess children's understanding; and ideas of ways to support children who are struggling as well as challenge children who are ready to move to the next level. In the planning sessions, the teacher and coach may also reflect on a plan that has already been developed by the teacher but check to make sure all of the elements mentioned are addressed. The teacher and coach determine when the lesson will be taught to arrange for the coach to observe.

For each observation the teacher and coach agree on one aspect of teacher practice to focus on during observation to enable them to dig deep into that practice to support teacher growth. The Erikson Institute Early Math Collaborative (2015) has identified nine dimensions organized around three domains that examine teacher practice. The first domain focuses on the WHAT and examines learning objectives, mathematical representations, and concept development. The second domain focuses on the WHO and examines teacher behaviors including attention to the developmental trajectories, response to individual student needs, and developmentally appropriate learning formats. The last domain focuses on the HOW and examines the delivery of the lesson including planning, student engagement, and establishing a community of learning. Each dimension includes essential questions that examine the quality of mathematics instruction in early childhood classrooms. The following table outlines the essential questions under each of the dimensions (Erikson Institute Early Math Collaborative, 2015, p. 2).

WHAT	**Learning Objectives** Does the teacher make the learning objective, or purpose, of the lesson clear to students? Is the learning objective focused on procedures or concepts? **Mathematical Representations** Are the mathematical representations accurate? Do they help students make sense of mathematical ideas? **Concept Development** Does the lesson lead students to a deeper understanding of concepts? Does the teacher help students generalize what they've learned?
WHO	**Attention to Developmental Trajectories** Is the lesson pitched to the right developmental level? Does the teacher scaffold to help build students' understanding? **Response to Students' Individual Needs** Is the teacher aware of how students are learning the concepts? Does the teacher adjust the lesson when necessary? **Developmentally Appropriate Learning Format** Is the format of the lesson developmentally appropriate?
HOW	**Planning** Has the teacher prepared for the lesson's activities? Is the teacher intentional in his/her plans or following a manual without making it his/her own? **Student Engagement** Who is doing the bulk of the math work – the thinking, explaining, and justification? **Establishment of a Mathematical Learning Community** Does the lesson promote a culture of learning where math is understood to be an important, sense-making activity? Are students welcomed to share their ideas?

During the observation the coach records both what the teacher is doing and what the children are doing throughout the lesson as well as comments and questions that arise during the observation. Several different forms of data collection can be used during the observation based on the preference of the teacher and coach and what method fits best with the focus of the observation. Possible data collection methods include video recording, audio recording, scripting, tallies or coding, shadowing one student, and examining student work. There is a great advantage to video recording because it allows the teacher and coach to watch the video together during the reflection phase.

The reflection phase occurs following the observation, either within the same visit or within a few days of the observation. The reflection conversation begins with a review of the teacher's goal and focus and the teacher's reflection on how it went in relationship to that specific goal and focus. The next step is to analyze the teacher data to examine teacher behaviors and how the data inform current or future practice. Next, the teacher and coach review the child data looking for evidence of learning and growth or evidence of confusion. It is also important to examine these data for the purpose of informing practice in future planning and instruction. The last piece of the reflection conversation focuses on what the teacher might do differently in the future as well as how to further explore the big idea presented in the observation lesson with the children.

Project Evaluation

To evaluate the effectiveness of the project, the participants will complete a pre- and post-test measure of participant knowledge and attitudes regarding mathematics, and an evaluation of participant satisfaction with the course. The Early Math Beliefs and Confidence Survey (EM-BCS) (Chen et al., 2014) is being used as the pre- and post-test measure (see Appendix M). On the EM-BCS participants use a five-point Likert scale to rate their degree of agreement or disagreement with 28 statements that assess three aspects of educator beliefs and confidence: 1) educators' beliefs about young children and mathematics; 2) educators' confidence in helping young children learn foundational mathematics knowledge and skills; and 3) educators' confidence in their own mathematics abilities (Chen et al., 2014). Participants complete the EM-BCS prior to the first professional development session and at the conclusion of the last session. The survey is completed anonymously and takes approximately 15 minutes to complete. The results of the survey are analyzed and used to answer the following questions about educators' beliefs and confidence:

- What do educators believe about teaching and learning mathematics in early childhood classrooms?
- How confident are educators in helping young children learn foundational mathematics knowledge and skills?
- How confident are educators in their own mathematics abilities?
- Does professional development focused on early childhood mathematics impact educators' attitudes and beliefs toward foundational mathematics?
- Does professional development focused on early childhood mathematics impact educators' confidence in helping young children learn foundational mathematics knowledge and skills?
- Does professional development focused on early childhood mathematics impact educators' confidence in their own mathematics abilities?

The results of this project evaluation will be used to support professional development that meets educators' needs in the area of early mathematics. The results may be disseminated on a local level to stakeholders involved in organizing and planning professional development opportunities for early childhood educators in Eastern Iowa. The stakeholders could include staff from the Area Education Agency, school districts, Early Childhood Iowa, Head Start, Child Care Resource and Referral, community colleges and universities, and private preschool or childcare programs. The results may also be shared at the state level with stakeholders from the Department of Education and area education agencies. The results may also be submitted for publication in an academic journal or presented at a scholarly conference.

In addition to the pre-post measure, participants are asked to complete a reflection log at the conclusion of each session to assess increase in their content knowledge and how the session informed their teaching. Participants rate their knowledge and skills related to the learning objectives prior to and after attending the session using a 4-point Likert scale. Participants are also asked to note teaching practices they will start doing, keep doing, and/or stop doing as a result of this session. The reflection also provides participants with the opportunity to note any additional supports or resources that are needed to put the knowledge and skills gained from this session into practice in the classroom. The session reflections can be found in the participant handouts for each session in the appendices (see Appendix B – J). A final course evaluation will be used to gather information on the application of the content, obstacles to using the information, as well as the quality of the training and the instructor (see Appendix N).

CONCLUSIONS AND RECOMMENDATIONS

Early mathematics plays an important role in building a foundation for young children's later learning. The depth and breadth of this course expands teachers' attitudes, knowledge, and skills in a way that supports them to build upon current practices and support children's thinking and learning. This course helps teachers intentionally plan experiences that focus on mathematics as well as highlight naturally occurring problems and situations that lend themselves to mathematics learning.

One insight gained from this process is the importance of the mathematics language in early childhood settings. In every content topic area explored throughout this course the value of intentional modeling and scaffolding the use of mathematically rich language with children is emphasized for teachers. The use of mathematical language when exploring the various content topic areas in the big ideas is congruent with the overarching goal of supporting children's overall language development during the preschool years.

Another insight gained from this process is the connection between the foundational mathematical skills found in the big ideas and the direct alignment to the standards and goals that guide teaching and learning in preschool classrooms across the State of Iowa. My in depth exploration of this topic made the connection between the big ideas of early mathematics and the Iowa Early Learning Standards (Early Childhood Iowa, 2012), the objectives for development and learning (Heroman et al., 2010), and the quality program standards very clear. This correlates with the recommendation in the NAEYC/NCTM joint position statement that emphasized the value of "well aligned systems of appropriate high-quality standards, curriculum, and assessment" (2002, p. 11). It is evident that implementing practices that support children's development of mathematical learning centered around the big ideas only supports the implementation of these standards and goals and overall, develops a solid foundation for children's mathematical learning as they move into the traditional K-12 education system.

Additional insights will be gained following the initial implementation of this course with a group of teachers. It will provide an opportunity to examine the changes in attitudes, knowledge and skills as a result of the professional development and coaching. The feedback from participants will be used to identify enhancements or changes regarding the content and delivery of the course for continued improvement.

The NAEYC/NCTM Joint Position Statement (2002) highlighted the importance of high quality professional development for both pre-service and in-service practitioners that support early mathematics as well as structures and processes for ongoing learning and planning. While this course provides a wealth of information on foundational mathematics skills and the site-based coaching provides ongoing support between sessions, it is important that teachers continue build upon the knowledge and skills gained through professional development. Continued support for teachers to translate knowledge and skills into classroom practice and improved learning for children is essential. Avenues that could be explored to meet this need are professional learning communities (DuFour, DuFour, Eaker, & Many, 2006) or a peer coaching model such as Teachers Learning & Collaborating (National Center on Quality Teaching and Learning, 2014), These structures provide teachers with opportunities to collaborate with other teachers to plan for mathematics learning focused on common goals. Conversations following lesson implementation offer ongoing opportunities for discussion regarding the strategies that worked or did not work in supporting children's mathematical learning.

The current delivery design for this course is 9 half-day sessions that are 4 hours each. This delivery design could be adjusted based on the needs of teachers and programs and the time constraints of professional development calendars within school districts if it was a barrier to teacher participation. The sessions could be broken into smaller chunks of content to fit within the time constraints as long as the site-based coaching is preserved to support the transfer of new knowledge and skills into practice.

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APPENDICES

Appendix A

Materials List

Session	Required Materials
All	☐ Flip chart paper ☐ Markers ☐ Pens or pencils ☐ Timer ☐ Post-It notes ☐ Highlighters
Session 1	☐ Collections of sorting materials (at least one for each table)
Session 2	☐ Dot cards ☐ *Anno's Counting Book* Big Book (Anno, 1992) ☐ *Anno's Counting Book* (Anno, 1975/1977) ☐ Unit blocks
Session 3	☐ Counting jar (one for each table) containing a small number of items ☐ Rekenrek attendance chart ☐ Popsicle sticks

Session 4	☐ Operations symbols to post including
	☐ *The Gingerbread Man* (Mackinnon, 2007)
	☐ Materials for games
	    ☐ Game of NIM – 15 pennies or counters
	    ☐ Achi – Achi game board, 4 counters of one color and 4 counters of another color
	    ☐ Balloon Volleyball – Balloon, way of recording tally marks
	    ☐ Itsy Bitsy Spider Game – Dot cube, egg carton without a lid, "spider" game piece such as plastic spider or spider ring
	    ☐ Matching Cards – Set of cards with matching numbers represented in two different ways (dot arrangements and numerals, for example)

Session 5	☐ A variety of materials for creating patterns ☐ Chart paper labeled with the following headings: children's artwork, blocks, songs and finger plays, instruments, outside, and books ☐ Unit blocks
Session 6	☐ Pairs of paper strips cut into various lengths (should be same width and color) ☐ Collections of miscellaneous materials for ordering
Session 7	☐ Unit blocks
Session 8	☐ Paper shapes ☐ Three-dimensional shapes
Session 9	☐ Cards with representing 1 through 25 in different ways (dots, tally marks, numerals, Roman numerals) ☐ Pattern blocks ☐ Each of the 26 big ideas of mathematics on a strip of paper

Appendix B

Note-Catcher for Session 1

... a place to jot down musings, questions, and ideas you want to remember ...

Setting the stages for our learning
Mathematics of sets & sorting
How children develop ideas about attributes

Note-Catcher Video Analysis: Sets & Sorting

Focus on the Child: Sets & Sorting

<u>Questions to Consider While Viewing the Video Clips</u>

- What Big Ideas about sets and sorting do these children seem to understand?
- What Big Ideas about sets and sorting do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring sets and sorting might a teacher provide for these children to encourage them to develop their understanding further?

Clip/Child		Notes about children's thinking
"Sorting Rocks"	Child 31	
	Child 14	
"Sorting Commercial Manipulatives"	Child 33	

Note-Catcher Video Analysis: Sets & Sorting

Lesson: People Sort

Some Questions to Consider While Viewing the Video Clip

Questions	Notes
<u>About the Children</u> • What Big Ideas about sets and sorting do these children seem to understand? • What Big Ideas about sets and sorting do these children seem to be learning? • What, specifically, do these children say or do that gives you evidence of their thinking?	

<u>About the Teacher</u>
• What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    • About materials to use?
    • About questions to ask?
    • About space arrangement?
• How does the teacher scaffold the children's thinking and explaining?	

<u>About the Activity</u> • How does the teacher tie the activity to the book *Five Creatures*? • What modifications might you make if you were doing this activity in your classroom? • What opportunities for exploring sets and sorting might a teacher provide for these children to encourage them to develop their understanding further?	

Reflection for Session 1
Setting the Stage & Sorting It All Out

Participants Name ______________________________ Email ______________________

Please evaluate your knowledge prior to attending this training session and after attending.
1 = I have no knowledge of this concept
2 = I have little knowledge of this concept
3 = I have some knowledge of this concept
4 = I have a lot of knowledge of this concept

Before attending...	Participant knowledge and skills	After attending...
1 2 3 4	Describe the importance of early mathematics and how this course supports teachers to develop children's foundational mathematics skills.	1 2 3 4
1 2 3 4	Summarize the big ideas of sets and sorting, how children's understanding of sets and sorting develops, and the implications for early childhood classrooms.	1 2 3 4

Because of this session, my teaching will be informed, and I will...

Start doing –

Keep doing –

Stop doing –

Additional supports/resources needed to put new knowledge and skills into practice –

Appendix C

Note-Catcher for Session 2

... a place to jot down musings, questions, and ideas you want to remember ...

Making sense of number sense
How children develop ideas about number
Where's the math in blocks?

Note-Catcher Video Analysis: Number Sense

Focus on the Child: Number Sense

Questions to Consider While Viewing the Video Clips

- What Big Ideas about number do these children seem to understand?
- What Big Ideas about number do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring number might a teacher provide for these children to encourage them to develop their understanding further?

Clip/Child		Notes about children's thinking
"Comparing Quantity"	Child 4	
"Recognizing Quantities"	Child 1	
	Child 2	
"Matching Quantities"	Child 3	

Note-Catcher Video Analysis: Number Sense

Lesson: Number Arrangements

Some Questions to Consider While Viewing the Video Clip

<table>
<tr><th>Questions</th><th>Notes</th></tr>
<tr><td><u>About the Children</u>
What Big Ideas about number do these children seem to understand?
What Big Ideas about number do these children seem to be learning?
What, specifically, do these children say or do that gives you evidence of their thinking?</td><td></td></tr>
<tr><td><u>About the Teacher</u>
What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
About materials to use?
About questions to ask?
About space arrangement?
How does the teacher scaffold the children's thinking and explaining?</td><td></td></tr>
<tr><td><u>About the Activity</u>
What does it do for their children's understanding to describe their arrangements using numbers?
What modifications might you make if you were doing this activity in your classroom?
What opportunities for exploring number might a teacher provide for these children to encourage them to develop their understanding further?</td><td></td></tr>
</table>

Learning Trajectory for Recognition of Number and Subitizing

Age (years)	Developmental Progression	Looks like:
0-1	Does not have intentional knowledge of number.	
1-2	Names groups of one to two.	Sees two shoes and says "Two shoes."
3	Nonverbally makes a small collection with the same number.	When shown a collection of three, makes another collection of three.
4	Instantly recognizes collections up to four briefly shown and verbally names the number of items.	When shown four objects briefly, says "four."
5	Instantly recognizes collections up to five briefly shown and verbally names the number of items.	Shown five objects briefly, says "five."
	Verbally labels all arrangements to about five when shown only briefly.	"I saw three and two and so I said five."
	Verbally labels most briefly shown arrangements to six, then up to 10.	"I made two groups of three and one more, so seven."
6	Verbally labels structured arrangements up to 20.	"I saw three 5's, so 5, 10, 15"
7	Verbally labels structured arrangements shown only briefly, using groups, place value, and skip counting.	"I saw groups of tens and twos so 10,20,30,40,42,44,46."
8	Verbally labels structured arrangements shown only briefly, using groups, multiplication, and place value.	"I saw groups of tens and threes, so five tens is 50 and four threes is 12, so 62 in all."

Adapted from: Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.)(pp. 17-20). New York, NY: Routledge.

Reflection for Session 2

Making Sense of Number Sense & Where's the Math in Blocks (Part 1)

Participants Name ______________________________ Email ______________________

Please evaluate your knowledge prior to attending this training session and after attending.

1 = I have no knowledge of this concept
2 = I have little knowledge of this concept
3 = I have some knowledge of this concept
4 = I have a lot of knowledge of this concept

Before attending...	Participant knowledge and skills	After attending...
1 2 3 4	Summarize the big ideas of number sense, how children's understanding of number sense develops, and the implications for early childhood classrooms.	1 2 3 4
1 2 3 4	Discuss the value of block play in early childhood classrooms and describe the stages of block play.	1 2 3 4

Because of this session, my teaching will be informed, and I will...

Start doing –

Keep doing –

Stop doing –

Additional supports/resources needed to put new knowledge and skills into practice –

Appendix D

Note-Catcher for Session 3

... a place to jot down musings, questions, and ideas you want to remember ...

Counting: More than just 1, 2, 3
How children develop ideas about counting
Math in routines

Note-Catcher Video Analysis: Counting

Focus on the Child: Counting

<u>Questions to Consider While Viewing the Video Clips</u>

- What Big Ideas about counting do these children seem to understand?
- What Big Ideas about counting do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring counting might a teacher provide for these children to encourage them to develop their understanding further?

<table>
<tr><th colspan="2">Clip/Child</th><th>Notes about children's thinking</th></tr>
<tr><td>"Learning Counting Words"</td><td>Child 4</td><td></td></tr>
<tr><td rowspan="2">"One-to-One Correspondence"</td><td>Child 6</td><td></td></tr>
<tr><td>Child 18</td><td></td></tr>
</table>

Note-Catcher Video Analysis: Counting

Lesson: Movement Counts

Some Questions to Consider While Viewing the Video Clip

Questions	Notes
About the Children • What Big Ideas about counting do these children seem to understand? • What Big Ideas about counting do these children seem to be learning? • What, specifically, do these children say or do that gives you evidence of their thinking?	

About the Teacher
• What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    • About materials to use?
    • About questions to ask?
    • About space arrangement?
• How does the teacher scaffold the children's thinking and explaining?	

About the Activity • What does it do for the children's understanding to connect numbers with movement? • What modifications might you make if you were doing this activity in your classroom? • What opportunities for exploring counting might a teacher provide for these children to encourage them to develop their understanding further?	

Learning Trajectory for Counting

Age (years)	Developmental Progression	Looks like:
1	No verbal counting	
	Chants "sign song" or sometimes-indistinguishable number words	Names some number words with no sequence.
2	Verbally counts with separate words, not necessarily in the correct order above five.	"One, two, three, four, five, seven."
	Puts objects, actions, and words in many-to-one or overly ridged one-to-one correspondence.	Counts two objects "two, two, two."
3	Verbally counts to ten with some correspondence with objects	
	Keeps one-to-one correspondence between counting and objects	
4	Accurately counts objects in a line to five and answers the "how many" question with the last number counted.	
	Counts arrangements of objects to 10. May also be able to write numerals 1-10.	Accurately counts a line of 9 blocks and says there are nine.
	May be able to state number after a number. Verbal counting to 20 is developing.	What comes after 4? "1, 2, 3, 4, 5. 5!"
	Counts out object to 5. Recognizes that counting is relevant to situations in which a certain number must be placed.	Produces a group of 4 objects.
5	Counts and counts out objects accurately to 10, then beyond (to about 30). Understands cardinality (how numbers tell how many) Recognizes errors in others counting and can eliminate errors in own counting (point-object) when asked to try hard.	Counts a scattered group of 19 chips, keeping track of them by moving each one as they are counted.
	Counts backward from 10 to 1, verbally, or when removing objects from a group.	"10, 9, 8, 7, 6, 5, 4, 3, 2, 1!"
6	Counts verbally and with objects from numbers other than 1.	Asked to "count from 5 to 8," counts, "5, 6, 7, 8!"
	Determines numbers just after or just before a number.	Asked, "What comes just before 7?" says, "Six!"
	Skip counts by 10s up to 100 or beyond with understanding.	"10, 20, 30…100!"

Age	Developmental Progression	Looks like:
6	Counts to 100. Makes decade transitions (e.g., from 29 to 30) starting at any number.	
	Keeps track of a few counting acts, by only using numerical pattern (spatial, auditory or rhythmic).	"How much is 3 more than 5?" child feels 3 "beats" as they count, "5…*6, 7, 8*!"
	Counts by fives or twos with understanding.	Child counts, "2, 4, 6, 8…30."
	Counts mental images of hidden objects.	Asked, "There are 5 chips hers and 4 under the napkin, how many in all?" Says, "Fiiiiive…,"then points to the napkin in 4 distinct points [corners of an imagined square] saying, "6, 7, 8, 9."
	Keeps track of counting acts numerically, first with objects, then by "counting counts." Counts up 1 to 4 *more* from a given number.	How many is 3 more than 6? "6…7 [puts up a finger], 8 [puts up another finger], 9 [puts up third finger]. 9."
	Understands the base ten numeration system and place value concepts.	Counts by tens and ones to determine.
	Understands value of a digit according to the place of he digit within a number.	Shown 3 whole plastic eggs and 4 halves, counts and says there are 5 whole eggs.
	Counts unusual units, such as "wholes" when shown combinations of wholes and parts.	
	Counts to 200 or beyond, recognizing the patterns of ones, tens, and hundreds.	
7	Consistently conserves number (i.e., believes the number has been unchanged), even in face of perceptual distractions such as the spreading out of objects in a collection.	Counts 2 rows that are laid out across from each other and says they are the same. Even when adult spreads one row out says, "Both still have the same number; one's just longer."
	Counts "counting words" (single sequence or skip counts) in either direction. Recognizes that decades sequence mirrors single digit sequence.	What's 4 less than 63? "62 is 1, 61 is 2, 60 is 3, 59 is 4…so, 59."
	Switches between sequence and composition views of multi-digit numbers easily.	Counts backwards form 20 and higher with meaning.

Adapted from: Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.) (pp. 36-48). New York, NY: Routledge.

Reflection for Session 3

More than 1, 2, 3 & Math in Routines

Participants Name ________________________ Email ____________________

Please evaluate your knowledge prior to attending this training Session and after attending.
1 = I have no knowledge of this concept
2 = I have little knowledge of this concept
3 = I have some knowledge of this concept
4 = I have a lot of knowledge of this concept

Before attending...	Participant knowledge and skills	After attending...
1 2 3 4	Summarize the big ideas of counting, how children's understanding develops, and ideas for exploring counting in the classroom	1 2 3 4
1 2 3 4	Discuss how to support mathematics within classroom routines.	1 2 3 4

Because of this session, my teaching will be informed, and I will...

Start doing –

Keep doing –

Stop doing –

Additional supports/resources needed to put new knowledge and skills into practice –

Appendix E

Note-Catcher for Session 4

... a place to jot down musings, questions, and ideas you want to remember ...

The story of number operations
How children develop ideas about number operations
Math in games

Note-Catcher Video Analysis: Number Operations

Focus on the Child: Number Operations

<u>Questions to Consider While Viewing the Video Clips</u>

- What Big Ideas about number operations do these children seem to understand?
- What Big Ideas about number operations do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring number operations might a teacher provide for these children to encourage them to develop their understanding further?

Clip/Child		Notes about children's thinking
"Changing Quantity"	Child 13	
"Comparing Quantity"	Child 14	
"Parts & Wholes"	Child 29	

Note-Catcher Video Analysis: Number Operations

Lesson: Mouse Collections

Some Questions to Consider While Viewing the Video Clip

Questions	Notes
About the Children • What Big Ideas about number operations do these children seem to understand? • What Big Ideas about number operations do these children seem to be learning? • What, specifically, do these children say or do that gives you evidence of their thinking?	

About the Teacher
• What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
  • About materials to use?
  • About questions to ask?
  • About space arrangement?
• How does the teacher scaffold the children's thinking and explaining?	

About the Activity • How does the teacher tie the activity to the book *Mouse Count*? • What modifications might you make if you were doing this activity in your classroom? • What opportunities for exploring number operations might a teacher provide for these children to encourage them to develop their understanding further?	

Learning Trajectory for Addition and Subtraction (Emphasizing Counting Strategies)

Age (years)	Developmental Progression	Looks like:
1	Sensitivity to adding and subtracting perceptually combined groups. No formal adding.	Shows no signs of understanding adding or subtracting.
2-3	Adds and subtracts very small groups nonverbally.	Shown 2 objects then 1 object going under a napkin, identifies or makes a set of 3 objects to "match".
4	Finds sums for joining problems up to 3 + 2 by "counting all" with objects.	Asked, "You have 2 balls and get 1 more. How many in all?," counts out 2, then counts out 1 more, then counts all 3: "1, 2, 3...3!"
4-5	Finds sums for joining ("You had 3 apples and get 3 more, how many do you have in all?") and part-part whole ("There are 6 girls and 5 boys on the playground, how many children were there in all?") problems by *direct modeling, counting all, with objects.*	Asked, "You have 2 red balls and 3 blue balls. How many in all?," counts out 2 red, then counts out 3 blue, then counts all 5.
	Solves take-away problems by separating with objects.	Asked, "You have 5 balls and give 2 to Tom. How many do you have left?," counts out 5 balls, then takes away 2, and then counts remaining 3.
	Adds on objects to "make one number into another," without needing to count from "one."	Asked, "This puppet has four balls but she should have six. Make it six," puts up four fingers on one hand, immediately count up from four while putting up two more fingers, saying, "Five, six."
	Finds the missing addend (5 + __ = 7) by adding on objects.	*Join-to—Count All-Groups*: Asked, "You have 5 balls and then get some more. Now you have 7 in all. How many did you get?," counts out 5, then counts those 5 again starting at 1, then adds more, counting "6, 7," then counts the balls added to find the answer, 2. (Some children may use their fingers, and attenuate the counting by using finger patterns.)

Age	Developmental Progression	Looks like:
4-5	Finds the missing addend (5 + _ = 7) by adding on objects.	*Separate-To—Count-All-Groups*: Asked, "Nita had 8 stickers. She gave some to Carmen. Now she has 5 stickers. How many did she give to Carmen?," counts 8 objects, separates until 5 remain, counts those taken away.
	Compares by matching in simple situations.	*Match—Count Rest.* Asked, "Here are 6 dogs and 4 balls. If we give a ball to each dog, how many dogs won't get a ball?," counts out 6 dogs, matches 4 balls to 4 of them, then counts the 2 dogs that have no ball.
5-6	Finds sums for joining ("You had 8 apples and get 3 more...) and part-part-whole (6 girls and 5 boys...) problems with finger patterns and/or by counting on.	Counting on: "How much is 4 and 3 more?" "Fourrrr...five, six, seven [*uses rhythmic or finger pattern to keep track*]. Seven!" Counting-up-to: May solve missing addend (3 + _ = 7) or compare problems by counting up; e.g., counts "4, 5, 6, 7" while putting up fingers; and then counts or recognizes the 4 fingers raised. Asked, "You have 6 balls. How many more would you need to have 8?," says, "Six, seven [*puts up first finger*], eight [*puts up second finger*]. Two!"
6	Has initial part-whole understanding. Solves all previous problem types using flexible strategies (may use some known combinations, such as 5 + 5 is 10). Sometimes can do start unknown (_+6=11), but only by trial and error.	Asked, "You had some balls. Then you get 6 more. Now you have 11 balls. How many did you start with?," lays out 6, then 3more, counts and gets 9. Puts 1 more with the 3...says 10, then puts 1 more. Counts up from 6 to 11, then recounts the group added, and says, "Five!"

Age	Developmental Progression	Looks like:
6-7	Recognizes when a number is part of a whole and can keep the part and whole in mind simultaneously; solves "start unknown" (__ + 4 = 9) problems with counting strategies.	Asked, "You have some balls, then you get 4 more balls, now you have 9. How many did you have to start with?," counts, putting up fingers: "Five, six, seven, eight, nine." Looks at fingers, and says "Five!"
	Uses flexible strategies and derived combinations (e.g., "7 + 7 is 14, so 7 + 8 is 15) to solve all types of problems. Can simultaneously think of three numbers within a sum, and can move part of a number to another, aware of the increase in one and the decrease in another.	Asked, "What's 7 + 8?," thinks: 7 + 8 → 7 + [7+1] → [7+7] + 1 = 14 + 1 = 15.
	May solve simple cases of multi-digit addition (sometimes subtraction) by incrementing tens and/or ones.	"What's 20 + 34?" Student uses connecting cube to count up 20, 30, 40, 50 plus four is 54.
7	Solves all types of problems with flexible strategies and known combinations.	Asked, "If I have 13 and you have 9, how could we have the same number?," says, "9 and 1 is 10, then 3 more to make 13. 1 and 3 is 4. I need 4 more!"
	Multi-digit may be solved by incrementing or combining tens and ones (latter not used for join, change unknown).	"What's 28 + 35?" Child thinks: 20..., 30, 40, 50; then 58, 59, 60, 61, 62, 63.

Adapted from: Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.)(pp, 84-92). New York, NY: Routledge.

Learning Trajectory for Comparing, Ordering, and Estimating

Age (years)	Developmental Progression	Looks like:
0-1	Puts objects, words, or actions in one-to-one or many-to-one correspondence or a mixture.	Puts several blocks in each muffin tin.
2	Puts objects in rigid one-to-one correspondence. Uses words to include "more", "less" or "same." Implicitly sensitive to the relation of more than/less than involving very small numbers.	Puts one block in each muffin tin, but is disturbed that some blocks left so finds more tine to put every last block in something.
	Puts objects into one-to-one correspondence, although may not fully understand that this creates equal groups.	Put a straw in each carton (doesn't worry if extra straws are left), but doesn't necessarily know there are the same numbers of straws and cartons.
	Compares collections that are quite different in size (e.g., one is at least twice the other).	Shown 10 blocks and 25 blocks, points to the 25 as having more.
	If the collections are similar, compares very small numbers. Compares collections using number words "one" and "two".	Shown groups of 2 and 4, points to the group of 4 as having more.
3	Identifies the "first" and often "second" objects in a sequence.	
	Compares collections of 1 to 4 items verbally or nonverbally ("just by looking"). The items must be the same. May compare the smallest collections using number words "two" and "three", and "three" and others. Can transfer an ordering relation from one pair of collections to another.	Identifies ••• and ••• as equal and different from •• or • •.
4	Matches small, equal collections, showing that they are the same number.	Matches collections of three shells and three dots and then declares that they "have the same number."
	Compares groups of 1 to 6 by matching.	Give one toy bone to every do and say there are the same number of dogs and bones.
4	Accurate comparison via counting, but only when objects are about the same size and groups are small (up to 5).	Counts two piles of 5 blocks each, and says they are the same.
	Not always accurate when larger group's objects are smaller in size than the objects in the smaller group.	Accurately counts two equal groups, but when asked, says the group of larger blocks has more.

Age (years)	Developmental Progression	Looks like:
4	Uses knowledge of counting number relationships to determine relative size and position when given perceptual support.	Shown a 0 at one end of a line and 5 at the other end, places a "3" approximately in the middle.
5	Compares with counting, even when larger collection's objects are smaller. Later, figures out how many more or less.	Accurately counts two equal collections, and says they have the same number, even if one collection has larger blocks.
	Identifies and uses ordinal numbers from "first" to "tenth".	Can identify what place in line.
	Names a "small number" (e.g., from 1 to 4) for sets that cover little space and a "big number" (10-20 or more) for sets that cover a lot of space.	Shown nine objects spread out for one second and asked "How many?" and responds "Fifty".
	Compares with counting, even when larger collection's objects are smaller, up to 10.	Accurately counts two groups of nine each, and says they have the same number, even if one collection has large blocks.
6	Uses internal images and knowledge of number relationships to determine relative size and position.	Which number is closer to 6: 4, or 9?
	Orders numerals in a collection (small numbers first). And a group of numerals with.	Given cards with one to five dots on them, puts in order.
	Orders lengths marked into units.	Given towers of cubes, puts in order, one to ten.
6	Extends sets and number categories to include "small numbers," which are usually subitized, not estimated, "middle size numbers" (e.g., 10-20) and "large numbers."	Shown 9 objects spread out for one second and asked "How many?," responds, "Fifteen".
7	Compares numbers with place value understandings	"63 is more than 59 because 6 tens is more than 5 tens even if there are more than 3 ones."
	Uses internal images and knowledge of number relationships, including ones embedded in tens, to determine relative size and position.	Asked "Which is closer to 45, 30, or 50?" Responds "45 is right next to 50, but 30 isn't."
	Numerosity Estimation	Shown 40 objects spread out for one second and asked "How many?" Responds "About thirty."

Age	Developmental Progression	Looks like:
8	Uses internal images and knowledge of number relationships, including place value, to determine relative size and position.	Asked "Which is closer to 3500, 2000 or 7000?" Responds "70 is double 35, but 20 is only 15 from 35, so 20 hundreds (2000) is closer".
	Initially, a portion of the to-be-estimated collection is counted; this is used as a benchmark from which an estimate is made. Later scanning can be linked to recalled benchmarks.	Shown 11, says "It looked closer to 10 than 20, so I guess 12." Shown 45 objects spread out for one second and asked "How many?" Responds "About five tens—fifty."
	Initially for regular arrangements, subitizing is used to quantify a subset and repeated addition or multiplication used to produce an estimate. Later, the process is extended to include irregular arrangements. Finally, it includes the ability to decompose or partition the to-be-estimated collection into convenient subset sizes, then recompose the numerosity based on multiplication.	Show 87 objects spread out and asked for an estimate responds, "That's about 20—so, 20, 40, 60, 80. Eighty.'"

Adapted from: Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.) (pp. 57-67). New York, NY: Routledge.

The Game of NIM

Goal: To reason quantitatively and abstractly in order to leave your opponent with the last counter

Materials: 15 pennies or counters

Number of Players: 2

Directions:

1. Spread out the 15 pennies of counters over the playing surface.
2. Take turns taking 1, 2, or 3 counters from the set.
3. The player who takes the last counter loses the game.

"Achi"

Goal: To demonstrate reasoning and thinking ahead in order to achieve 3 counters in a row

Materials: Achi game board, 4 counters of one color and 4 counters of another color

Number of Players: 2

Directions: This game is played like tic-tac-toe. Each player has 4 counters.

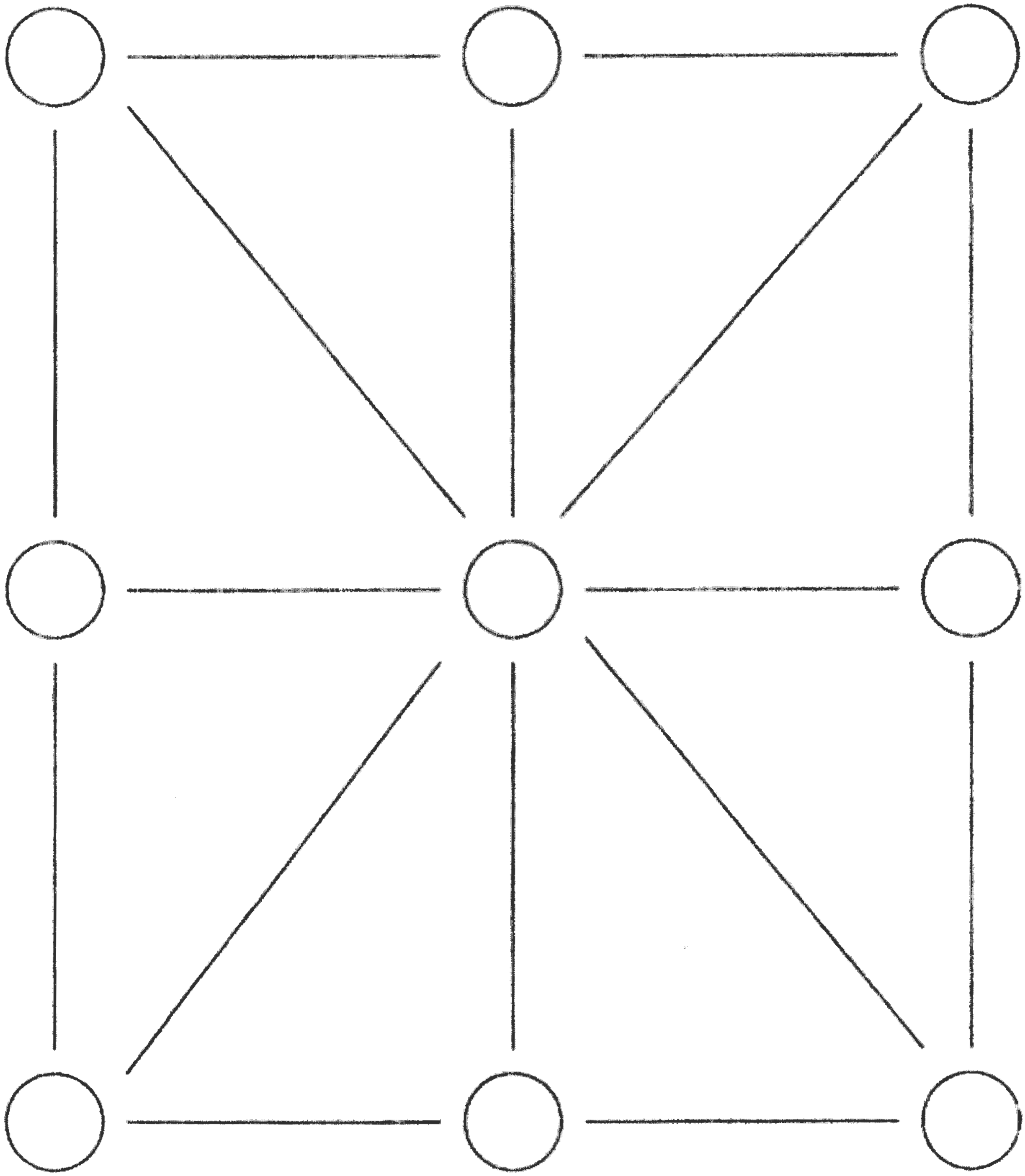
1. Taking turns, each player places one counter on a circle.

2. If three counters of one color are in a row, that player calls out "Achi" and wins the round.

3. If both players have played all their pieces and no one has won, then, players take turns sliding a counter on a line to the empty place.

4. When a player makes a line of three of his/her counters by sliding, that player calls out "Achi" and wins the round.

5. The game can be played over and over again for many round so that both players will win and develop strategic thinking.



Balloon Volleyball

Goal: To count using one-to-one correspondence to determine number of balloon taps and to compare number of taps for each round

Materials: Balloon, way of recording tally marks

Number of Players: 2 groups of children (up to whole group)

Directions:

1. Children should sit on the floor divided into two groups. Have each group arrange themselves in rows like on a volleyball court.

2. Toss a balloon into the center of the two groups. Children try to keep the balloon up in the air by tapping it with their fingers. Children must remain sitting the entire time.

3. Teacher (and children) counts each balloon tap and makes a tally mark for every tap.

4. Children try to make as many taps as possible before the balloon touches the floor. Play at least three times. Compare the number of taps for each game to determine the best strategy for balloon volleyball.

Itsy Bitsy Spider Game

Goal: To count spaces on a game board with one-to-one correspondence and to subitize small quantities of dots

Materials: Dot cube, egg carton without a lid, "spider" game piece such as plastic spider or spider ring

Number of Players: One or two players

Directions:

1. Place the egg carton vertically so that it represents the waterspout going up and down. You may want to draw arrows up one side and down the other to indicate the spider's path.

2. Children take turns rolling the dot cube, subitizing (or counting) the number of dots, and then moving their spiders that number of spaces—first up and then down the "spout."

3. Ask children to decide together whether the spiders need to land exactly in the last cup. Play ends when both spiders complete the path.

Memory

Goal: To associate number names, quantities, and written numerals

Materials: Set of cards with matching numbers represented in two different ways (dot arrangements and numerals, for example)

Number of Players: Two players

Directions:

1. Place dot cards facedown in one row and the numeral cards facedown in another row.

2. Children take turns turning over one dot card and one numeral card. They say the number name for each card. If the cards match, the player keeps the cards. If the cards do not match, they are turned facedown again.

3. Players take turns until all the cards are matched.

Variations:
You can introduce this game in stages by having children play with all cards face-up at first, and then with one set (dots or numerals) face-up and the other set face-down. Some children may also benefit from matching numeral cards to numeral cards and/or dot cards to dot cards.

Vary the cards. Use five-frame or ten-frame cards, for example.

Use matching cards to play Go Fish!

Reflection for Session 4
The Story Behind Operations & Math in Games

Participants Name ________________________________ Email ________________________

Please evaluate your knowledge prior to attending this training Session and after attending.

1 = I have no knowledge of this concept
2 = I have little knowledge of this concept
3 = I have some knowledge of this concept
4 = I have a lot of knowledge of this concept

Before attending...	Participant knowledge and skills	After attending...
1 2 3 4	Summarize the big ideas of number operations, how children's understanding of number operations develops, and ideas for exploring counting in the classroom.	1 2 3 4
1 2 3 4	Discuss how games support children's mathematical learning.	1 2 3 4

Because of this session, my teaching will be informed, and I will...

Start doing –

Keep doing –

Stop doing –

Additional supports/resources needed to put new knowledge and skills into practice –

Appendix F

Note-Catcher for Session 5

... a place to jot down musings, questions, and ideas you want to remember ...

Recognizing repetition and regularity in patterns
How children develop ideas about patterns
Where is the math in blocks (part 2)

Note-Catcher Video Analysis: Patterns

Focus on the Child: Patterns

Questions to Consider While Viewing the Video Clips

- What Big Ideas about patterns do these children seem to understand?
- What Big Ideas about patterns do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring patterns might a teacher provide for these children to encourage them to develop their understanding further?

Clip/Child		Notes about children's thinking
"Growing Patterns"	Child 32	
	Child 30	
"Repeating Patterns"	Child 32	

Note-Catcher Video Analysis: Patterns

Lesson: Who is Napping?

Some Questions to Consider While Viewing the Video Clip

Questions	Notes
<u>About the Children</u> • What Big Ideas about patterns do these children seem to understand? • What Big Ideas about patterns do these children seem to be learning? • What, specifically, do these children say or do that gives you evidence of their thinking?	

<u>About the Teacher</u>
• What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    • About materials to use?
    • About questions to ask?
    • About space arrangement?
• How does the teacher scaffold the children's thinking and explaining?	

<u>About the Activity</u> • How does this activity help children think about patterns? • What modifications might you make if you were doing this activity in your classroom? • What opportunities for exploring patterns might a teacher provide for these children to encourage them to develop their understanding further?	

Learning Trajectory for Patterns and Structure

Age (years)	Developmental Progression	Looks like:
2	Detects and uses patterning implicitly, but may not recognize sequential linear patterns explicitly or accurately.	Names a striped shirt with not repeating unit a "pattern."
3	Recognizes a simple pattern.	"I'm wearing a pattern" about a shirt with black, white, black, white (and so on) stripes.
4	Fills in missing element of pattern, first with ABAB patterns.	Given objects in a row with one missing, ABAB_BAB, identifies and fills in the missing element.
	Duplicates ABABAB pattern. May have to be close to the pattern.	Given objects in a row, ABABAB, makes their own ABABAB row in a different location.
	Extends AB repeating patterns.	Given objects in a row, ABABAB, adds ABAB to the end of row.
	Duplicates simple patterns (not just along side the model pattern).	Given objects in a row, ABBABBABB, makes their own ABBABBABB in a different location.
5	Extends simple repeating patterns.	Given objects in a row, ABBABBABB, adds ABBABB to the end of the row.
6	Identifies the smallest unit of a pattern. Can translate patterns into new media.	Given objects in a row, ABBABBABB, identifies the core unit of the pattern as "ABB."
7	Describes a pattern numerically, can translate between geometric and numeric representation of a series.	Given objects in a geometric pattern, describes the numeric progression.

Adapted from: Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.) (pp. 221-225). New York, NY: Routledge.

Reflection for Session 5
Recognizing Repetition and Regularity & Math in Blocks (Part 2)

Participants Name ______________________ Email ______________________

Please evaluate your knowledge prior to attending this training Session and after attending.
1 = I have no knowledge of this concept
2 = I have little knowledge of this concept
3 = I have some knowledge of this concept
4 = I have a lot of knowledge of this concept

Before attending...	Participant knowledge and skills	After attending...
1 2 3 4	Summarize the big ideas of pattern, how children's understanding of pattern develops, and ideas for exploring pattern in the classroom.	1 2 3 4
1 2 3 4	Discuss the relationships between blocks and how blocks can be used to support children's mathematical thinking.	1 2 3 4

Because of this session, my teaching will be informed, and I will...

Start doing –

Keep doing –

Stop doing –

Additional supports/resources needed to put new knowledge and skills into practice –

Appendix G

Note-Catcher for Session 6

... a place to jot down musings, questions, and ideas you want to remember ...

What kind of "big" is it
How children develop ideas about measurement
Questions and answers with data analysis
How children develop ideas about data analysis

Note-Catcher Video Analysis: Measurement

Focus on the Child: Measurement

Questions to Consider While Viewing the Video Clips

- What Big Ideas about measurement do these children seem to understand?
- What Big Ideas about measurement do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring measurement might a teacher provide for these children to encourage them to develop their understanding further?

Clip/Child		Notes about children's thinking
"Comparing Objects by Length"	Child 20	
	Child 23	
	Child 22	
"Seriating Objects by Length & Capacity"	Child 14	
	Child 20	

Note-Catcher Video Analysis: Measurement

Lesson: Just Right for Me

Some Questions to Consider While Viewing the Video Clip

Questions	Notes
<u>About the Children</u> • What Big Ideas about measurement do these children seem to understand? • What Big Ideas about measurement do these children seem to be learning? • What, specifically, do these children say or do that gives you evidence of their thinking?	

<u>About the Teacher</u>
• What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    • About materials to use?
    • About questions to ask?
    • About space arrangement?
• How does the teacher scaffold the children's thinking and explaining?	

<u>About the Activity</u> • What does it do for the children's understanding to describe how they knew it was just right for them? • What modifications might you make if you were doing this activity in your classroom? • What opportunities for exploring measurement might a teacher provide for these children to encourage them to develop their understanding further?	

Learning Trajectory for Length Measurement

Age (years)	Developmental Progression	Looks like:
2	Does not identify length as an attribute.	"This is long. Everything straight is long. It it's not straight, it can't be long."
3	Identifies length/distance as an attribute. May understand length as an absolute descriptor (e.g., all adults are tall), but not as a comparative (e.g., one person is taller than another). May compare not-corresponding parts of a shape in determining length.	"I'm tall, see?"
4	Physically aligns two objects to determine which is longer or if they are the same length.	Stand two sticks up next to each other on a table and says, "This one is bigger."
4	Compares length of two objects with a third object. When asked to measure, may assign a length by guessing or moving along a length while counting. May be able to measure with a ruler, but often lacks understanding or skill (e.g., ignores starting point).	Compares length of two objects with a piece of string. Moves finger along a line segment, saying, "10, 20 30, 31, 32." Measures two objects with a ruler to check if they are the same length, but does not accurately set the "zero point" for one of the items.
5-6	Orders lengths, marked in 1 to 6 units.	Given towers of cubes, puts in order: 1 to 6.
	Lays units end to end. May not recognize the need for equal-length units. The ability to apply resulting measures to comparison situations develops later at this level.	Lays 9 1-inch cubes in a line beside a book to measure how long it is.
7	Measures by repeated use of a unit (but initially may not be precise in such iterations). Relates sizes and number of units explicitly (but may not appreciate the need for identical units in every situation). Can add up two lengths to obtain the length of a whole. Iterates a single unit to measure. Recognizes that different units will result in different measures and that identical units should be used.	"If you measure with centimeters instead inches, you'll need more of them, because each one is smaller." "This is five long and this one is three long, so they are eight long together." Measures a book's length accurately with a ruler.

Age (years)	**Developmental Progression**	**Looks like:**
8	Considers the length of a bent path as the sum of its parts (not the distance between end points). Measures knowing needs for identical units, relationships between different units, etc. Begins to estimate.	"I used a meter stick three times, then there was a little left over. So, I lined it up from 0 and found 14 centimeters.
8	Possesses and "internal" measurement tool. Mentally moves along an object, segmenting it, and counting the segments. Operates arithmetically on measured lengths ("connected lengths"). Estimates with accuracy.	"I imagine one meter stick after another along the edge of the room. That's how I estimated the room's length is 9 meters."

Adapted from: Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.) (pp. 194-197). New York, NY: Routledge.

Learning Trajectory for Volume Measurement

Age (years)	Developmental Progression	Looks like:
0-3	Identifies capacity or volume as an attribute.	Says, "This box holds a lot of blocks!"
4-5	Can compare two containers. Compares objects by physically or mentally aligning; refers to at least two dimensions of objects. May be able to compare two containers using a third container.	Fills a container using another (smaller container) and counts the number needed to completely fill the larger container. Pours one container into two others, concluding that one holds less because it overflows, and the other is not fully filled.
6	Partial understanding of cubes as filling a space. Able to estimate the number of scoops needed to fill. Able to attend to both the portion of container filled and the portion remaining empty. Compares objects by physically and mentally aligning and explicitly recognizing three dimensions.	Initially, may count the faces of a cube building, possibly double-counting cubes at the corners and usually not counting internal cubes. Eventually counts one cube at a time in carefully structured and guided contexts such a packing a small box
7	Uses simple units to fill containers, with accurate counting. Relates size and number of units explicityly; understands that fewer larger than smaller units will be needed to pack or fill a container. Can accurately convert units in 1:2 ratio.	Fills a container by repeatedly filling a unit and counting how many. With teaching, understands that fewer larger than smaller objects or units will be needed to fill a given container.
7	Understands cubes as filling a space, but does not use layers of multiplicative thinking. Moves to more accurate counting strategies.	Counts unsystematically, but attempts to account for internal cubes. Counts systematically, trying to account for outside and inside cubes. Counts the numbers of cubes in one row or column of a 3-D structure and using skip counting to get the total.

Age (years)	Developmental Progression	Looks like:
8	Able to coordinate flexibly filling, packing, building aspects of volume. Initially counts or computes (e.g., number of rows times number of columns) the number of cubes in one layer, and then uses addition or skip counting by layers to determine the total volume. Eventually moves to multiplication (e.g., number of cubes in a layer time the number of layers).	Counts or computes (row b column) the number of cues in one row, and then uses addition or skip counting to determine the total. Computes (row times column) the number of cubes in one row, and then multiplies by the number of layers to determine the total
9	Has an abstrct understanding of the rectangular prism volume formula. With linear measures or other similar indication of the two dimensions, multiplicatively iterates squares in a row or column to determine the area.	Can compute the volume of rectangular prisms from its dimensions and explain how that multiplication creates a measure of volume.

Adapted from: Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.) (pp. 207-208). New York, NY: Routledge.

Note-Catcher Video Analysis: Data Analysis

Focus on the Child: Data Analysis

Questions to Consider While Viewing the Video Clips

- What Big Ideas about data analysis do these children seem to understand?
- What Big Ideas about data analysis do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring data analysis might a teacher provide for these children to encourage them to develop their understanding further?

Clip/Child		Notes about children's thinking
"Age Chart"	Child 27	
"Attendance Chart"	Child 27	
	Child 15	

Note-Catcher Video Analysis: Data Analysis

Lesson: Shoe Graph

Some Questions to Consider While Viewing the Video Clip

Questions	Notes
<u>About the Children</u> • What Big Ideas about data analysis do these children seem to understand? • What Big Ideas about data analysis do these children seem to be learning? • What, specifically, do these children say or do that gives you evidence of their thinking?	

<u>About the Teacher</u>
• What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    • About materials to use?
    • About questions to ask?
    • About space arrangement?
• How does the teacher scaffold the children's thinking and explaining?	

<u>About the Activity</u> • How does this activity help children think about data analysis? • What modifications might you make if you were doing this activity in your classroom? • What opportunities for exploring data analysis might a teacher provide for these children to encourage them to develop their understanding further?	

Reflection for Session 6
What Kind of "Big" is It? & Asking Questions and Finding Answers

Participants Name ______________________________ Email ______________________

Please evaluate your knowledge prior to attending this training Session and after attending.

1 = I have no knowledge of this concept
2 = I have little knowledge of this concept
3 = I have some knowledge of this concept
4 = I have a lot of knowledge of this concept

Before attending...	Participant knowledge and skills	After attending...
1 2 3 4	Summarize the big ideas of measurement, how children's understanding of measurement develops, and ideas for exploring measurement in the classroom.	1 2 3 4
1 2 3 4	Summarize the big ideas of data analysis, how children's understanding of data analysis develops, and ideas for exploring measurement in the classroom.	1 2 3 4

Because of this session, my teaching will be informed, and I will...

Start doing –

Keep doing –

Stop doing –

Additional supports/resources needed to put new knowledge and skills into practice –

Appendix H

Note-Catcher for Session 7

...a place to jot down musings, questions, and ideas you want to remember ...

<u>Making sense of the world around us through spatial relationships</u>
<u>How children develop ideas about spatial relationships</u>
<u>Where is the math in blocks? (part 3)</u>

Note-Catcher Video Analysis: Spatial Relationships

Focus on the Child: Spatial Relationships

Questions to Consider While Viewing the Video Clips

- What Big Ideas about spatial relationships do these children seem to understand?
- What Big Ideas about spatial relationships do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring spatial relationships might a teacher provide for these children to encourage them to develop their understanding further?

Clip/Child		Notes about children's thinking
"Describing Relative Location"	Child 22	
	Child 35	
"Directional Prepositions"	ELL	

Note-Catcher Video Analysis: Spatial Relationships

Lesson: Walk with Rosie

Some Questions to Consider While Viewing the Video Clip

Questions	Notes
<u>About the Children</u> • What Big Ideas about spatial relationships do these children seem to understand? • What Big Ideas about spatial relationships do these children seem to be learning? • What, specifically, do these children say or do that gives you evidence of their thinking?	

<u>About the Teacher</u>
• What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    • About materials to use?
    • About questions to ask?
    • About space arrangement?
• How does the teacher scaffold the children's thinking and explaining?	

<u>About the Activity</u> • How does this activity help children think about spatial relationships? • What modifications might you make if you were doing this activity in your classroom? • What opportunities for exploring spatial relationships might a teacher provide for these children to encourage them to develop their understanding further?	

Learning Trajectories for Spatial Thinking

Age (years)	Developmental Progression	Looks Like:
a. Spatial Orientation		
0-2	Uses a distance landmark to find an object or location near it, if they have not personally moved relative to the landmark. Understands initial vocabulary of spatial relations and location.	Understands initial vocabulary of spatial relations and location.
2-3	Uses distant landmarks to find objects or location near them, even after they have moved themselves relative to the landmarks, if the target object is specified ahead of time.	Orients a horizontal or vertical line in space.
4	Locates objects after movement, even if target is not specified ahead of time. Searches a small area comprehensively, often using a circular search pattern.	Extrapolates lines from positions on both axes and determines where they intersect in meaningful contexts.
5	Locates objects after movement (relates several locations separately from own position), maintaining the overall shape of the arrangement of objects. Represents objects' positions relative to landmarks (e.g., about half way in between two landmarks) and keeps track of own location in open areas or mazes. Some use coordinate in simple situations.	
6	Locates objects using maps with pictorial cues.	Can extrapolate two coordinates, understanding the integration of them to one position, as well as use coordinate labels in simple situations.
7	Reads and plots coordinates on maps.	
8+	Follows a simple route map, with more accurate direction and distances. Uses general frameworks that include the observer and landmarks. May not use precise measurement even when that would be helpful, unless guided to do so.	Can follow and create maps, even if spatial relations are transformed.

Age (years)	Developmental Progression	Looks Like:
b. Spatial Visualization and Imagery		
0-3	Can move shapes to a location.	
4	Mentally turns object in easy tasks.	Given a shape with the top marked with color, correctly identifies which of the three shapes it would look like if it were turned "like this" (90 degree turn demonstrated) before physically moving he shape.
5	Uses the correct motions, but not always accurate in direction and amount.	Knows a shape has to be flipped to match another shape, but flips it in the wrong direction.
6	Performs slides and flips, often only horizontal and vertical, using manipulatives. Performs turns of 45, 90, and 180 degrees.	Knows that a shape must be turned a certain amount to fit into a puzzle.
7	Performs diagonal slides and flips.	Knows how much an object needs to be turned and flipped over an oblique line (45 degree orientation) to fit into a puzzle.
8+	Predicts results of moving shapes using mental images.	"If you turned this 120 degrees, it would be just like this one."

Adapted from: Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.) (pp. 137-140). New York, NY: Routledge.

Reflection for Session 7
Mapping the World Around Us & Math in Blocks (Part 3)

Participants Name ______________________ Email ______________________

Please evaluate your knowledge prior to attending this training Session and after attending.

1 = I have no knowledge of this concept
2 = I have little knowledge of this concept
3 = I have some knowledge of this concept
4 = I have a lot of knowledge of this concept

Before attending...	Participant knowledge and skills	After attending...
1 2 3 4	Summarize the big ideas of spatial relationships, how children's understanding of spatial relationships develops, and ideas for exploring spatial relationships in the classroom.	1 2 3 4
1 2 3 4	Describe how blocks support children's mathematical thinking and how teachers can scaffold children's thinking during block building experiences.	1 2 3 4

Because of this session, my teaching will be informed, and I will...

Start doing –

Keep doing –

Stop doing -

Additional supports/resources needed to put new knowledge and skills into practice –

Appendix I

Note-Catcher for Session 8

... a place to jot down musings, questions, and ideas you want to remember ...

The shape of things

How children develop ideas about shape

Good math in good books

Note-Catcher Video Analysis: Shape

Focus on the Child: Shape

Questions to Consider While Viewing the Video Clips

- What Big Ideas about shape do these children seem to understand?
- What Big Ideas about shape do these children seem to be learning?
- What, specifically, do these children say or do that gives you evidence of their thinking?
- What opportunities for exploring shape might a teacher provide for these children to encourage them to develop their understanding further?

Clip/Child		Notes about children's thinking
"Recognizing Shapes"	Child 10	
	Child 11	
"Sorting Geo-Solids"	Child 5	
"Composing Shapes"	Child 12	
	Child 15	

Note-Catcher Video Analysis: Shape

Lesson: Feel For Shapes

Some Questions to Consider While Viewing the Video Clip

Questions	Notes
About the Children • What Big Ideas about shape do these children seem to understand? • What Big Ideas about shape do these children seem to be learning? • What, specifically, do these children say or do that gives you evidence of their thinking?	

About the Teacher
• What kind of instructional decisions has the teacher made in terms of the logistics of the activity?
    • About materials to use?
    • About questions to ask?
    • About space arrangement?
• How does the teacher scaffold the children's thinking and explaining?	

About the Activity • How does this activity help children think about shape? • What modifications might you make if you were doing this activity in your classroom? • What opportunities for exploring shape might a teacher provide for these children to encourage them to develop their understanding further?	

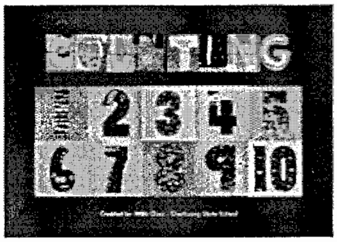
Learning Trajectory for Shapes

Age (years)	Developmental Progression	Looks like:
0-2	Compares real world objects.	Says two pictures of houses are the same or different.
	Matches familiar shapes (circle, square, rectangle, triangle) with *same size and orientation.*	Matches square to an identical square.
	Matches familiar shapes with *different sizes.*	Matches a big square to a small square.
	Matches familiar shapes with *different orientations.*	Matches a square to a rhombus (diamond).
3	Recognizes and names typical circle, square, and less often, triangle. May physically rotate shapes in atypical orientations to mentally match them to a prototype.	Names this a "square": 
	Judges two shapes the same if they are more visually similar than different	"These are the same because they are pointy at the top."
3-4	Matches wider variety of shapes with the *same size and orientation.*	
	Matches a wider variety of shapes with *different sizes and orientations.*	Matches rectangles that are big, little, fat, thin, vertical, horizontal.
	Matches *combinations* of shapes to each other.	Matches overlapping circles.
4	Recognizes some less typical squares and triangles, and may recognize some rectangles, but usually not rhombuses (diamonds). Often doesn't differentiate sides/corners.	Names triangles that do not have three equal length sides all triangles.
	Says two shapes are the same after matching one side on each	When presented triangles matching on one side, says "These are the same."
4	Uses manipulatives representing parts of shapes, such as sides, to make a shape that "looks like" a goal shape. May think of angles as a corner (which is "pointy").	Makes a triangle using sticks.
	Looks for differences in attributes, but may examine only part of shape.	"These are the same" (indicating the top halves of the f the shapes are similar because they are both "pointy" at the top).

Age (years)	Developmental Progression	Looks like:
4-5	Recognizes rectangle sizes, shapes, and orientations of rectangles. concept of a right angle is more firmly embedded	Correctly labels big, little, fat, thin, sideways rectangles.
	Identifies sides as distinct geometric objects.	When shown an arrow shape says, "This is a quadrilateral because it has four straight sides."
	Looks for differences in attributes, examining full shapes, but may ignore some spatial relationships.	
	Recognizes angles as separate geometric objects, at least in the limited context of "corners"	Asked why this is a triangle, says "It has three angles" and counts them, pointing clearly at each vertex (point at the corner).
5	Recognizes most familiar shapes and typical examples of other shapes.	Names hexagon, rhombus, trapezoid, etc.
6	Names most common shapes without making mistakes. Recognizes (at least) right angles.	Avoids calling an oval a circle. Distinguishes between rectangle and parallelogram without right angles.
7	Can recognize and describe contexts in which angle knowledge is relevant.	Uses descriptors such as sharper, bent, bend in road, crossings.
7	Identifies shapes in terms of their components.	Says "No matter how skinny it looks, that's a triangle because it has three sides and three angles."
	Determines congruence by comparing all attributes and all spatial relationships.	Says that two shapes are the same shape and the same size after comparing every one of their sides and angles.
	Moves and places objects on top of each other to determine congruence.	Says that two shapes are the same shape and the same size because they can be laid on top of each other.
	Uses manipulatives representing parts of shapes, such as sides and angle "connectors," to make a shape that is completely correct, based on knowledge of components and relationships.	Asked to make a triangle with sticks, creates this: 

Age (years)	Developmental Progression	Looks like:
8+	Represents various angle contexts as two lines, explicitly including the reference line; and, at least implicitly, the size of the angle as the rotation between these lines.	
	Refers to geometric properties and explains with transformations.	"These must be 'congruent,' because they have equal sides, all square corners, and I can move them on top of each other exactly."
	Uses class membership (e.g., to sort), not explicitly based on properties.	"I put the triangles over here, and the quadrilaterals, including squares, rectangles, rhombuses, and trapezoids, over there."
	Uses properties explicitly. Can see the invariants in the changes of state or shape, but maintaining the shapes' properties.	"I put the shapes with opposite sides parallel over here, and those with four sides but not both pairs of sides parallel over here."
8+	Uses class membership for shapes (e.g., to sort or consider shapes "similar") explicitly based on properties, including angle and measure. Is aware of restriction of transformations and also of the definitions and can integrate the two. Sorts hierarchically, based on properties.	"I put the 'equilateral triangles' over her, and "scalene triangles' over her. The 'isosceles triangles' are all these... they included the equilaterals."
	Combines various meanings of angle (turn, corner, slant), including angle measure.	"This ramp is at a 45 degree angle to the ground."

Adapted from: Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.)(pp. 157-169). New York, NY: Routledge.



Here are some questions to help you analyze the mathematics of a counting book:

- *How high does the book count to? (1-10, higher)*
- *As the numbers change, are the illustrations cumulative (more join or leave the same group) or is each number a new set of things?*
- *Are any hierarchical relationships depicted in the illustrations (one smaller number embedded in a larger number)?*
- *Is zero used appropriately?*
- *If involving higher numbers, does the book introduce patterns or arrays or somehow reinforce the idea of grouping and place value?*
- *Does the book introduce separating and joining concepts (addition and subtraction)? Does it do so at fairly simple level of counting up or down by ones? Are counting-on strategies possible?*
- *Is the counting embedded in a story that helps make a math all around us connection?*
- *Is the counting tied to another informational concept, such as animal study?*

Finding Good Math in Good Books

Title: ______________________________

Author: ______________________________

What's the Math?

Identify specific BIG IDEAS of Math that this book might be well suited to use to introduce or develop understanding for the children in your classroom. Please be as specific as possible. (*For example, instead of stating "number sense" identify "composing and decomposing numbers up to ten," "making reasonable estimates," or ...*)

Big Ideas in this Book

What are some open-ended discussion questions or problem situations I can ask that will trigger mathematical thinking or understanding?

(For example, every monkey in Caps for Sale has one hat. Which picture makes it easier to count how many - the one of the monkeys in the tree or the one of the man sleeping with his hats piled on his head? Why?)

What are some extending activities that will allow children to develop and construct mathematical understanding?

(For example, after reading Five Creatures, have children decide how many creatures live in their house and then draw and label a picture modeled after one in the book - collect into a classroom book.)

Reflection for Session 8
The Shape of Things & Good Math in Good Books

Participants Name ______________________ Email ______________________

Please evaluate your knowledge prior to attending this training Session and after attending.

1 = I have no knowledge of this concept
2 = I have little knowledge of this concept
3 = I have some knowledge of this concept
4 = I have a lot of knowledge of this concept

Before attending...	Participant knowledge and skills	After attending...
1 2 3 4	Summarize the big ideas of shape, how children's understanding of shape develops, and ideas for exploring shape in the classroom.	1 2 3 4
1 2 3 4	Describe characteristics to consider in counting books and how books support children's mathematical thinking.	1 2 3 4

Because of this session, my teaching will be informed, and I will...

Start doing –

Keep doing –

Stop doing -

Additional supports/resources needed to put new knowledge and skills into practice –

Appendix J

Note-Catcher for Session 9

... a place to jot down musings, questions, and ideas you want to remember ...

Making connections among the Big Ideas of early mathematics
What is mindset? How does mindset affect math thinking and learning?

Reflection for Session 9
Big Connections – Math & Mindset

Participants Name ________________________________ Email ________________________

Please evaluate your knowledge prior to attending this training Session and after attending.

1 = I have no knowledge of this concept
2 = I have little knowledge of this concept
3 = I have some knowledge of this concept
4 = I have a lot of knowledge of this concept

Before attending...	Participant knowledge and skills	After attending...
1 2 3 4	Describe connections between the big ideas and the connections to classroom practices.	1 2 3 4
1 2 3 4	Define mindset and describe how it affects learning and growth for both teachers and children particularly in relation to mathematics.	1 2 3 4

Because of this session, my teaching will be informed, and I will...

Start doing –

Keep doing –

Stop doing -

Additional supports/resources needed to put new knowledge and skills into practice –

Appendix K

Resources for Mathematics in Early Childhood

Online Resources

- Erikson Early Math Collaborative - http://earlymath.erikson.edu
  - Facebook - https://www.facebook.com/earlymath
  - Pinterest - https://www.pinterest.com/earlymath/
  - Twitter - https://twitter.com/eriksonmath?lang=en
  - YouTube - https://www.youtube.com/user/eriksonmath/videos
- Institute of Education Sciences What Works Clearinghouse- http://ies.ed.gov/ncee/wwc/
  - Teaching Math to Young Children Practice Guide - http://ies.ed.gov/ncee/wwc/pdf/practice_guides/early_math_pg_111313.pdf
- National Association for the Education of Young Children - www.naeyc.org
  - Position Statement page - http://www.naeyc.org/positionstatements/mathematics
  - Position Statement PDF - http://www.naeyc.org/files/naeyc/file/positions/psmath.pdf
  - Executive Summary PDF- http://www.naeyc.org/files/naeyc/file/positions/Mathematics_Exec.pdf
  - NAEYC for Families - Support Math with Materials in Your Home - http://families.naeyc.org/learning-and-development/music-math-more/support-math-materials-your-home
  - NAEYC for Families - 5 Ways to Build Math into Your Child's Day - http://families.naeyc.org/learning-and-development/music-math-more/5-ways-build-math-your-childs-day
- National Council of Teachers of Mathematics - http://www.nctm.org
  - Position Statement PDF - http://www.nctm.org/Standards-and-Positions/Position-Statements/Mathematics-in-Early-Childhood-Learning/

Books

Chalufour, I., & Worth, K. (2004). *Building structures with young children*. St. Paul, MN: Redleaf Press.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: the learning trajectories approach* (2nd ed.). New York, NY: Routledge.

Copley, J. V. (2010). *The young child and mathematics* (2nd ed.). Washington, DC: National Association for the Education of Young Children & National Council of Teachers of Mathematics.

Editors of Teaching Young Children. (2015). *Exploring math and science in preschool.* Washington, DC: National Association for the Education of Young Children.

Erikson Institute Early Math Collaborative. (2014). *Big ideas of early mathematics: What teachers of young children need to know.* Boston, MA: Pearson.

Hirsch, E. S. (Ed.). (1996). *The block book* (3rd ed.). Washington, DC: National Association for the Education of Young Children.

Resources for Mathematics in Early Childhood (continued)

Moomaw, S., & Hieronymus, B. (2011). *More than counting: Math activities for preschool and kindergarten*. St. Paul, MN: Redleaf Press.

Newburger, A., & Vaughan, E. (2006). *Teaching numeracy, language, and literacy with blocks*. St. Paul, MN: Redleaf Press.

Pollman, M. J. (2010). *Blocks and beyond: Strengthening early math and science skills through spatial learning*. Baltimore, MD: Brookes Publishing.

Shillady, A. (Ed.). (2012). *Spotlight on young children: Exploring math*. Washington, DC: National Association for the Education of Young Children.

Appendix L

Children's Books Highlighting Mathematical Concepts

Title	Genre	Mathematical Concept	Synopsis	Citation
Anno's Counting Book	Wordless	Number Sense	A landscape changes over times of day, seasons, months, and years.	Anno, M. (2007). *Anno's counting book.* New York, NY: HarperCollins. (Original work published 1975)
Up, Down and Around	Narrative	Spatial Relationships	Positional words are introduced in the context of gardening.	Ayers, K. (2008). *Up, down and around.* (Illus. N. B. Wescott). Cambridge, MA: Candlewick Press. (Original work published 2007)
Ten, Nine, Eight	Narrative	Counting	A rhythmic count down to bedtime as a little girl and her father go through a bedtime routine.	Bang, M. (2003). *Ten, nine, eight.* New York, NY: HarperCollins. (Original work published 1985)
Yellow Ball	Narrative	Spatial Relationships	A yellow ball drifts out to sea.	Bang, M. (2016). *Yellow ball.* Cynthiana, KY: Purple House Press. (Original work published 1991)
Building a House	Informational	Spatial Relationships	Simple words and pictures are used to explain how a house is built.	Barton, B. (1990). *Building a house.* New York, NY: HarperCollins.
Hippos Go Berserk	Narrative	Number Operations	A lonely hippo calls two friends and the result is a boisterous hippo party.	Boynton, S. (1996). *Hippos go berserk.* New York, NY: Aladdin.
1, 2, 3 to the Zoo	Wordless	Number Sense	Train cars carry animals to the zoo.	Carle, E. (1999). *1, 2, 3 to the zoo.* New York, NY: The Trumpet Club. (Original work published 1968)
Rooster's Off to See the World	Narrative	Patterns	As Rooster goes off to explore the world, he is joined by groups of animals that increase by one each time.	Carle, E. (1999). *Rooster's off to see the world.* New York, NY: Aladdin. (Original work published 1972)
Woof! Woof!	Narrative	Shapes	Cutout shapes are used to create familiar animals in a guessing game.	Carter, D. (2006). *Woof! woof!* New York, NY: Little Simon.
Whoo? Whoo?	Narrative	Shapes	Cutout shapes create familiar animals in a guessing game.	Carter, D. (2007). *Whoo? whoo?* New York, NY: Little Simon.
Where Do I Live?	Narrative	Spatial Relationships	A gradual exploration of where children live	Chesanow, N. (1995). *Where do I live?* (Illus. A. Iosa). Hauppauge, NY: Barron's Educational Series.

Title	Genre	Mathematical Concept	Synopsis	Citation
Five Little Monkeys Jumping on the Bed	Narrative	Counting	Some little monkeys engage in shenanigans at bedtime resulting in bumps and bruises.	Christelow, E. (2004). *Five little monkeys jumping on the bed.* New York, NY: Clarion Books. (Original work published 1989)
Big Bug	Informational	Measurement	A book of comparisons showing size is relative.	Cole, H. (2014). *Big bug.* New York, NY: Little Simon.
Ten Black Dots	Narrative	Counting	Ten black dots are creatively used in different arrangements in each illustration.	Crews, D. (1995). *Ten black dots.* New York, NY: HarperCollins.
From Here to There	Narrative	Spatial Relationships	A young girl travels from her home to the street, town, country, and eventually the universe.	Cuyler, M. (1999). *From here to there.* (Illus. Y. C. Pak). New York, NY: Henry Holt and Company.
Ten in the Bed	Narrative	Counting	One child and nine stuffed animals roll out of bed.	Dale, P. (2006). *Ten in the bed.* London, England: Walker Books. (Original work published 1990)
Tangram Tales: Story Theater Using the Ancient Chinese Puzzle	Narrative	Shapes	A series of tales from around the world that can be told using tangrams.	de Las Casas, D. (2008). *Tangram tales: Story theater using the ancient Chinese puzzle.* Santa Barbara, CA: Libraries Unlimited.
The Shape of Things	Narrative	Shapes	Shapes are hidden in illustrations.	Dodds, D. A. (1994). *The shape of things.* (Illus. J. Lacome). Cambridge, MA: Candlewick Press.
Color Zoo	Wordless	Shapes	Shapes in die-cut pages display zoo animals as each page is turned.	Ehlert, L. (1989). *Color zoo.* New York, NY: HarperCollins.
Color Farm	Wordless	Shapes	Shapes in die-cut pages display farm animals as each page is turned.	Ehlert, L. (1990). *Color farm.* New York, NY: HarperCollins.
Fish Eyes: A Book You Can Count On	Narrative	Counting	An underwater counting adventure.	Ehlert, L. (1992). *Fish eyes: A book you can count on.* Boston, MA: Houghton Mifflin Harcourt.
This Jazz Man	Narrative	Counting	Introduces nine jazz musicians in the counting format of "This Old Man."	Ehrhardt, K. (2006). *This jazz man.* (Illus. R. G. Roth). Boston, MA: Houghton Mifflin Harcourt.
Feast for 10	Narrative	Counting	A family prepares for a feast.	Falwell, C. (1993). *Feast for 10.* New York, NY: Clarion Books.

Title	Genre	Mathematical Concept	Synopsis	Citation
When a Line Bends... A Shape Begins	Informational	Shapes	Rhymes describe how ten different shapes begin with a simple line.	Gowler Greene, R. (1997). *When a line bends... A shape begins.* (Illus. J. Kaczman). Boston, MA: Houghton Mifflin Harcourt.
The Village of Round and Square Houses	Narrative	Spatial Relationships	The story of village in Central Africa in which the women live in round houses and the men live in square ones and how this came to be.	Grifalconi, A. (1986). *The village of round and square houses.* Boston, MA: Little, Brown Books.
Pattern Bugs	Narrative	Patterns	Patterns are introduced in both graphics and text.	Harris, T. (2001). *Pattern bugs.* (Illus. A. C. Green). Minneapolis: MN: Millbrook Press.
Count and See	Wordless	Number Sense & Counting	Displays a picture of sets found all around us paired numeral, number word, and corresponding number of dots.	Hoban, T. (1972). *Count and See.* New York, NY: Simon & Schuster.
I Read Signs	Wordless	Shapes	Thirty familiar signs invite children to discover the concept of shape.	Hoban, T. (1987). *I read signs.* New York, NY: HarperCollins. (Original work published 1983)
Shapes, Shapes, Shapes	Wordless	Shapes	Photographs display different attributes and shapes found all around.	Hoban, T. (1996). *Shapes, shapes, shapes.* New York, NY: Mulberry Books. (Original work published 1986)
Is It Larger? Is It Smaller?	Wordless	Sets	Photographs depict attributes found all around.	Hoban, T. (1997). *Is it larger? Is it smaller?* New York, NY: Greenwillow Books. (Original work published 1985)
More, Fewer, Less	Wordless	Number Operations	Everyday object and familiar animals illustrate quantitative concepts.	Hoban, T. (1998a). *More, fewer, less.* New York, NY: HarperCollins.
So Many Circles, So Many Squares	Wordless	Shapes		Hoban, T. (1998b). *So many circles, so many squares.* New York, NY: Greenwillow Books.
Cubes, Cones, Cylinders, & Spheres	Wordless	Shapes	Three-dimensional shapes are highlighted in photographs of everyday scenes.	Hoban, T. (2000). *Cubes, Cones, Cylinders, & Spheres.* New York, NY: Greenwillow Books.

Title	Genre	Mathematical Concept	Synopsis	Citation
Rosie's Walk	Narrative	Spatial Relationships	Rosie the hen leaves the chicken coop and goes for a walk followed by a sly fox.	Hutchins, P. (1986). *Rosie's walk*. New York, NY: Aladdin. (Original work published 1968)
Changes, Changes	Wordless	Spatial Relationships	A set of building blocks transforms into an exciting adventure.	Hutchins, P. (1987). *Changes, changes*. New York, NY: Aladdin.
The Doorbell Rang	Narrative	Number Operations	A mathematical problem arises when mom makes a batch of cookies and the number of children there to eat them increases.	Hutchins, P. (1989). *The doorbell rang*. New York, NY: HarperCollins.
Five Creatures	Narrative	Sets & Number Operations	A little girl describes the similarities and differences between the creatures in her family.	Jenkins, E. (2001). *Five creatures*. (Illus. T. Bogacki). New York, NY: Square Fish.
Actual Size	Informational	Measurement	A look at the length, weight, and other interesting details about creatures.	Jenkins, S. (2004). *Actual size*. Boston, MA: Houghton Mifflin Harcourt.
Prehistoric Actual Size	Informational	Measurement	A look at the actual sizes of creatures in the prehistoric world.	Jenkins, S. (2005). *Prehistoric actual size*. Boston, MA: Houghton Mifflin Harcourt.
Splash!	Narrative	Number Sense	Critters splash in and out of a backyard pond.	Jonas, A. (1995). *Splash!* New York, NY: HarperCollins.
The Carrot Seed	Narrative	Measurement	A boy plants a carrot seed and even though everyone tells him it won't grow, he tends it and is rewarded.	Krauss, R. (2004). *The carrot seed*. (Illus. C. Jackson). New York, NY: HarperCollins. (Original work published 1945)
The Growing Story	Narrative	Measurement	A look at measurement through the little changes in a boy as he grows.	Krauss, R. (2007). *The growing story*. (Illus. P. Rowand). New York, NY: HarperCollins. (Original work published 1947)
Inch by Inch	Narrative	Measurement	In inchworm keeps from being eaten by measuring several birds.	Lionni, L. (1995). *Inch by inch*. New York, NY: HarperCollins. (Original work published 1960)
Brown Bear, Brown Bear, What Do You See?	Narrative	Patterns	A brown bear sees many colorful creatures. Identifying the words that repeat "mathematizes" the patterns in the text.	Martin, B., Jr. (1992). *Brown bear, brown bear, what do you see?* (Illus. E. Carle). New York, NY: Henry Holt and Company. (Original work published 1967)

Title	Genre	Mathematical Concept	Synopsis	Citation
One Gorilla: A Counting Book	Narrative	Counting	A gorilla wanders through a variety of settings being counted along with other creatures.	Morouzumi, A. (1993). *One gorilla: A counting book.* New York, NY: Square Fish Books.
Houses and Homes	Informational	Spatial Relationships	Photographs depict homes different types of homes throughout the world.	Morris, A. (1995). *Houses and homes.* (Illus. K. Heyman). New York, NY: HarperCollins.
Shoes, Shoes, Shoes	Informational	Sets	Photographs display shoes from around the world organized into categories.	Morris, A. (1998). *Shoes, shoes, shoes.* New York, NY: HarperCollins. (Original work published 1995)
Tikki Tikki Tembo	Narrative	Measurement	The classic tale of a boy with a very long name is in danger.	Mosel, A. (1985). *Tikki tikki tembo.* (Illus. B. Lent). New York, NY: Scholastic. (Original work published 1968)
Beep Beep, Vroom Vroom!: Patterns	Narrative	Patterns	A young girl plays with her brother's toy cars and arranges them in various patterns.	Murphy, S. J. (2000). *Beep beep, vroom vroom!: Patterns.* (Illus. C. L. Demarest). New York, NY: HarperCollins.
Goldilocks and the Three Bears	Narrative	Sets	The classic tale retold.	Parragon Books. (2012). *Goldilocks and the three bears.* (G. Scott). Bath, United Kingdom: Author.
The Three Little Pigs	Narrative	Sets	The classic tale retold.	Parragon Books. (2012). *The three little pigs.* (Illus. M. Matsuoka). Bath, United Kingdom: Author.
Piggies in the Pumpkin Patch	Narrative	Spatial Relationships	Two little piggies all over the farm. Prepositions of direction are used to describe their adventure.	Peterson, M. (2010). *Piggies in the pumpkin patch.* (Illus. J. Rofé). Watertown, MA: Charlesbridge.
One is a Snail, Ten is a Crab: A Counting by Feet Book	Informational	Number Operations	A counting book looking at the number of feet different animals have.	Pulley Sayre, A., & Sayre, J. (2006). *One is a snail, ten is a crab: A counting by feet book.* (Illus. R. Cecil). Cambridge, MA: Candlewick Press.
10 Minutes till Bedtime	Narrative	Counting	A countdown to a child's bedtime lead by a family of hamsters.	Rathman, P. (1998). *10 minutes till bedtime.* New York, NY: G.P. Putnam Sons.
Next to an Ant	Narrative	Measurement	A cumulative story comparing sizes.	Rockliff, M. (2011). *Next to an ant.* (Illus. P. Constantin). New York, NY: Scholastic.

Title	Genre	Mathematical Concept	Synopsis	Citation
One Duck Stuck: A Mucky Ducky Counting Book	Narrative	Counting	Duck gets stuck in the muck and groups of marshland creatures come to offer help.	Root, P. (2003). *One duck stuck: A mucky ducky counting book.* (Illus. J. Chapman). Cambridge, MA: Candlewick Press. (Original work published 1989)
We're Going on a Bear Hunt	Narrative	Spatial Relationships	A family goes on a bear hunt and goes over, under, through and around various obstacles in this classic chant.	Rosen, M. (1989). *We're going on a bear hunt.* (Illus. H. Oxenbury). London, England: Walker Books.
Whose Shoes Are These?: A Look at Workers' Footwear – Slippers, Sneakers, and Boots	Informational	Data Analysis	Describes why certain types of footwear are helpful to the people who wear them.	Salas, L. P. (2006). *Whose shoes are these?: A look at workers' footwear – slippers, sneakers, and boots.* (Illus. A. B. Muehlenhardt). North Mankato, MN: Capstone Press.
Caps for Sale: A Tale of a Peddler, Some Monkeys and Their Monkey Business	Narrative	Sets & Counting	The classic tale of the peddler whose hats are stolen by mischievous monkeys.	Slobodokina, E. (1989). *Caps for sale: A tale of a peddler, some monkeys, and their monkey business.* New York, NY: Scholastic. (Original work published 1940)
Which Would You Rather Be?	Narrative	Data Analysis	A rabbit poses the question and offers interesting sets of choices to a boy and girl.	Steig, W. (2002). *Which would you rather be?* (Illus. H. Bliss). New York, NY: HarperCollins.
Block City	Narrative	Spatial Relationships	This version of the classic poem depicts the structures a young boy builds with blocks.	Stevenson, R. L. (2005). *Block city.* (Illus. D. Kirk). New York, NY: Simon & Schuster.
Mouse Count	Narrative	Counting	A snake finds some sleepy mice and puts them in a jar for a delicious meal. The mice get away when the snake wants just one more.	Stoll Walsh, E. (1991). *Mouse count.* Boston, MA: Houghton Mifflin Harcourt.
Mouse Shapes	Narrative	Shapes	Three clever mice use shapes to make something to trick a sneaky cat.	Stoll Walsh, E. (2007). *Mouse shapes.* Boston, MA: Houghton Mifflin Harcourt.
Me on the Map	Informational	Spatial Relationships	A young girl shows herself on maps beginning in her room and expanding to space.	Sweeney, J. (1996). *Me on the map.* (Illus. A. Cable). New York, NY: Dragonfly Books.

Title	**Genre**	**Mathematical Concept**	**Synopsis**	**Citation**
Lots and Lots of Zebra Stripes: Patterns in Nature	Informational	Patterns	Photographs display the many patterns found in nature.	Swineburne, S. (2002). *Lots and lots of zebra stripes: Patterns in nature.* Honesdale, PA: Boyds Mills Press. (Original work published 1998)
Whose Shoes?: A Shoe for Every Job	Informational	Data Analysis	Photographs show the right type of shoe for various occupations.	Swineburne, S. (2011). *Whose shoes?: A shoe for every job.* Honesdale, PA: Boyds Mills Press.
Round is a Mooncake: A Book of Shapes	Narrative	Shapes	A little girl discovers shapes in her urban neighborhood.	Thong, R. (2000). *Round is a mooncake: A book of shapes.* (Illus. G. Lin). San Francisco, CA: Chronicle Books.
Just a Little Bit	Narrative	Measurement	A minuscule mouse and an enormous elephant encounter a problem when the want to play on the seesaw.	Tompert, A. (1996). *Just a little bit.* (Illus. L. Munsinger). Boston, MA: Houghton Mifflin Harcourt. (Original work published 1988)
Grandfather Tang's Story	Narrative	Shapes	A little girl and her grandfather use tangrams to tell stories about two fox fairies that can change shape.	Tompert, A. (1990). *Grandfather Tang's Story.* (Illus. R. A. Parker). New York, NY: Dragonfly Books.
Over in the Meadow: A Traditional Counting Rhyme	Narrative	Counting & Number Operations	Animal babies obediently follow their mothers' directions as the sun shines over the meadow.	Voce, L. (1994). *Over in the meadow: A traditional counting rhyme.* Cambridge, MA: Candlewick Press.
I Went Walking	Narrative	Patterns	A little girl goes walking and meets many animals along the way. Identifying the words that repeat "mathematizes" the patterns in the text.	Williams, S. (1992). *I went walking.* (Illus. J. Vivas). Boston, MA: Houghton Mifflin Harcourt. (Original work published 1989)
A Frog in the Bog	Narrative	Counting	A frog swallows most of the insects in the bog and his belly grows and grows.	Wilson, K. (2003). *A frog in the bog.* (Illus. J. Rankin). New York, NY: Margaret K. McElderry Books.
The Napping House	Narrative	Patterns	A granny, a child, and several critters nap on a rainy day.	Wood, A. (1984.) *The napping house.* (Illus. D. Wood). Orlando, FL: Harcourt Brace.

Appendix M

Early Math Beliefs and Confidence Survey

Section 1: Beliefs About Preschoolers and Math

Below are some ideas shared by preschool teachers about preschoolers and math. Below, please indicate what you think about these ideas.

For each of the following statements, rate your agreement by checking the appropriate box.

Most children **in my class**:	*Strongly Disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly Agree*
1. Enter preschool with little math knowledge.					
2. Have the cognitive abilities to learn math.					
3. Should be helped to learn math in preschool.					
4. Are very interested in learning math.					
5. Need to learn math in preschool to be ready for kindergarten.					
6. Learn a great deal about math through their everyday activities.					
7. Need structured preschool math instruction.					
8. Should be helped to learn math using a published math curriculum.					

Section 2: Confidence in Helping Preschoolers Learn Math

Some preschool teachers report they don't feel comfortable helping preschoolers learn math. Others feel confident; still others say they are confident in some areas of math but not in others. Below, please indicate how you feel helping preschoolers learn math.

For each of the following statements, rate your agreement by checking the appropriate box.

I am confident in my **knowledge** of:	*Strongly Disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly Agree*
1. What children in my classroom know about math when they enter my classroom.					
2. Reasonable math goals for preschoolers.					
3. The best practices and strategies for helping preschoolers learn math.					
4. Local or national math standards for preschoolers.					
5. The best ways to assess children's math knowledge and understanding throughout the year.					

Early Math Beliefs and Confidence Survey (continued)

I am confident about my **ability** to:	*Strongly Disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly Agree*
6. Observe what preschoolers know about math.					
7. Incorporate math learning into common preschool situations (i.e. dramatic play).					
8. Plan activities to help preschoolers learn math.					
9. Further preschoolers' math knowledge when they make spontaneous math comments/ discoveries. *Example: When child says, "I have four blocks" asking child how many blocks he would have if you gave him one more.*					
10. Make sense of preschoolers' confusions when they learn math. *Example: Why a child thinks two triangles that are displayed in different rotations aren't the same shape.*					
11. Translate assessment results into curriculum plans.					

Section 3: Confidence in Your Math Abilities

Some preschool teachers report that they just aren't good at math. Others say they love math. Still others say how they feel depends on the specific area of math. In this section, please indicate how you feel about math and your math abilities.

For each of the following statements, rate your agreement by checking the appropriate box.

	Strongly Disagree	*Disagree*	*Neutral*	*Agree*	*Strongly Agree*
1. Math was one of my best subjects in school.					
2. Just the word "math" can make me nervous.					
3. I'm not a "math person".					
4. I can easily rotate objects in my mind *Example: Figuring out how something would look from another angle.*					
5. I like coming up with creative ways to solve math problems.					
6. I can easily convert fractions into percentages and decimals numbers.					
7. I have a bad sense of direction.					
8. I'm good at looking at numeric data and finding patterns.					
9. I'm good at estimating how tall something is or the distance between two locations.					

Thank you for completing this survey.

Survey developed by Chen, McCray, Adams, & Leow, 2014

Early Math Beliefs and Confidence Survey (continued)

Section 4: Additional Information

What is the highest degree or level of education you have completed?

High school graduate (includes equivalency)	
Some college, no degree	
Associate's degree	
Bachelor's degree	
Master's degree	
Doctorate degree	

How many years of experience do you have working with preschool aged children?

1-4 years	
5-10 years	
Over 10 years	

How is your preschool program structured?

One half-day preschool class	
Two half-day preschool classes	
Full-day preschool class	

In your educational experience, did you complete math methods classes?

Yes	
No	

How many hours of math professional development have you participated in over the last three years?

Less than 1 hour	
1 hour	
2 hours	
3 hours	
4 hours	
5 hours	
More than 5 hours	

Thank you for completing this survey.

Appendix N

Course Evaluation

Early Mathematics in Early Childhood Classrooms: An Exploration of the Big Ideas

Participants Name ______________________________ Email ______________________

Please evaluate the impact of this course on your attitudes and beliefs, teacher knowledge and skills, and classroom practices. Circle the number that corresponds with your evaluation.

1 = Strongly disagree 2 = Disagree 3 = Uncertain 4 = Agree 5 = Strongly Agree

This course improved my attitudes and beliefs about early mathematics.	1	2	3	4	5
This course improved my knowledge and skills regarding early mathematics.	1	2	3	4	5
This course improved my ability to improve my classroom practices to support children's development of foundational mathematics knowledge and skills.	1	2	3	4	5

Describe the aspects of this course that have been easy to apply in your work with children.

Describe the obstacles you have encountered in using the information gained from this course.

Please evaluate the quality of the course and the instructor. Circle the number that corresponds with your evaluation.

1 = Strongly disagree 2 = Disagree 3 = Uncertain 4 = Agree 5 = Strongly Agree

The instructor was knowledgeable of the course content.	1	2	3	4	5
The instructor was prepared for each training session.	1	2	3	4	5
The content of the training met my personal and professional needs.	1	2	3	4	5
The content of the training was adequate to build my knowledge of early mathematics.	1	2	3	4	5
The content of the training was applicable to my work with children.	1	2	3	4	5
I would recommend this course to others.	1	2	3	4	5

Additional comments related to the skills of the instructor or the content presented.